	1 (20p)	2 (20p)	3 (20p)	4 (20p)	5 (20p)	6 (10p bonus)	Total
PHYS122 2023-2024 Spring 1st Midterm, Time: 90 minutes							

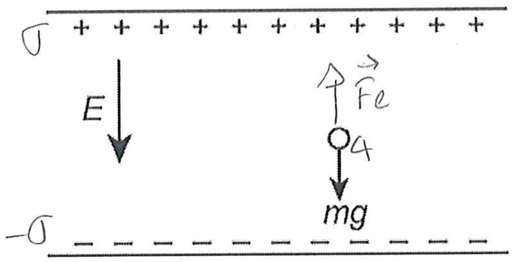
Name Surname: .... Student No: ....

Lecturer:...

You can use calculator during the exam, but exchanging is not allowed. Take ϵ_0=8,854x10^{-12} F/m

Good luck

1. A drop of oil, the mass of which is 1.00×10^{-4} g, is suspended in a uniform electric field (between two oppositely charged infinite plates). The electric field is 100 N/C in the downward direction. (a) Find the charge (including its sign) of the oil drop. (b) Find the surface charge density σ of the positively charged plate. (Take $g = 10.0\ \mathrm{m/s^2}$)



(a) $\vec{F_e} = q\vec{E}$ **5 points**

q must be negative

$|\vec{F_e}| = mg$ **5 points**

$|q| E = mg$

$$|q| = \frac{mg}{E} = \frac{1\times10^{-7}\,\mathrm{kg}\cdot 10\,\frac{m}{s^2}}{100\,\frac{N}{C}}$$

$|q| = 10^{-8}$ C **5 points**

$q = -1{,}00\times10^{-8}$ C

(b) $E = \frac{\sigma}{\epsilon_0}$

$$\sigma = \epsilon_0 E = 8{,}854\times10^{-12}\,\frac{C^2}{Nm^2}\cdot 100\,\frac{N}{C}$$ **5 points**

$$\sigma = 8{,}854\times10^{-10}\,\frac{C}{m^2}$$

2. An insulating sphere of radius R has a nonuniform charge density $\rho(r) = Ar^2$, where A is a constant.

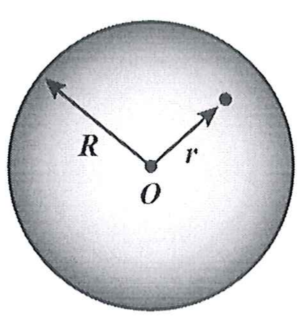
(a) If ρ is in $\frac{\mathrm{C}}{\mathrm{m}^3}$, and r is in m, find the unit of A .

(b) Find the total charge $Q = \int_0^R \rho(r) 4\pi r^2 dr$ of the sphere in terms of A and R.

(c) Find the magnitude of the electric field for $r > R$.

(d) Find the magnitude of the electric field for $r \leq R$.

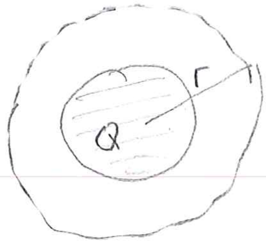
(e) Find the electric potential at the center of the sphere O.



(a) $A = \frac{\rho}{r^2}$, unit $= \frac{C/m^3}{m^2} = \frac{C}{m^5}$ 2 points

(b) $Q = \int_0^R Ar^2 4\pi r^2 dr = 4\pi A \int_0^R r^4 dr = \frac{4\pi A R^5}{5}$ 2 points

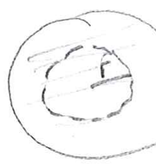
(c) $\Phi = E 4\pi r^2 = \frac{Q}{\epsilon_0}$



$E 4\pi r^2 = \frac{\frac{4\pi A R^5}{5}}{\epsilon_0}$ 4 points

$E = \frac{AR^5}{5\epsilon_0 r^2}$ $(r > R)$

(d) $\Phi = E 4\pi r^2 = \frac{1}{\epsilon_0} Q_{encl} = \frac{1}{\epsilon_0} \int_0^r \rho(r') 4\pi r'^2 dr'$



$E 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^r 4\pi A (r')^4 dr' = \frac{4\pi A r^5}{5\epsilon_0}$ 6 points

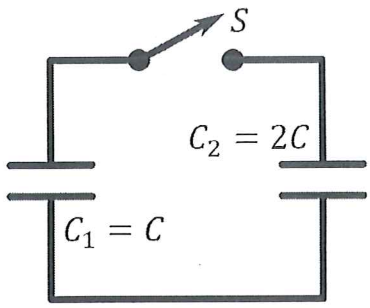
$E = \frac{Ar^3}{5\epsilon_0}$ $r \leq R$

(e) $V_0 = \int_0^\infty \vec{E} \cdot d\vec{l} = \int_0^\infty E\, dr = \int_0^R \frac{Ar^3}{5\epsilon_0} dr + \int_R^\infty \frac{AR^5}{5\epsilon_0 r^2} dr$

$V_0 = \left[\frac{Ar^4}{20\epsilon_0}\right]_0^R + \left[-\frac{AR^5}{5\epsilon_0 r}\right]_R^\infty = \frac{AR^4}{20\epsilon_0} + \frac{AR^4}{5\epsilon_0} = \frac{5AR^4}{20\epsilon_0} = \frac{AR^4}{4\epsilon_0}$

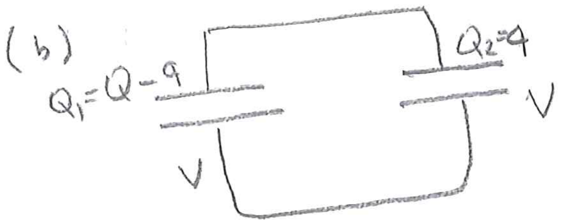
6 points (4)

4. A capacitor of capacitance $C_1 = C$ is charged to Q and then connected to another initially uncharged capacitor of capacitance $C_2 = 2C$, as shown in the figure. The switch S is initially in the open position. After S is closed, the system comes to equilibrium, that is to say, the potential difference and charges on the capacitors reach their final values. (a) Find the initial energy and (b) the final energy stored in the system. (c) How much energy is lost when the capacitors are connected?



(a) $E_{initial} = \frac{Q^2}{2C_1} = \frac{Q^2}{2C}$ [5 points]

(b)



$V = \frac{Q-q}{C_1} = \frac{q}{C_2}$

$\frac{Q-q}{C} = \frac{q}{2C}$ [5 points]

$Q = \frac{3q}{2}$

$q = 2Q/3$

$Q_1 = \frac{Q}{3}$, $Q_2 = \frac{2Q}{3}$

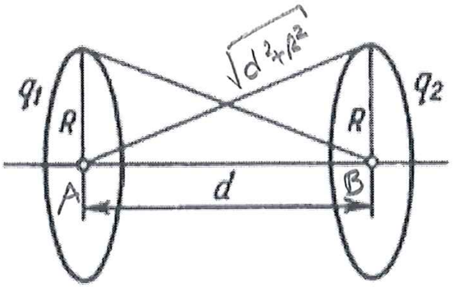
$E_{final} = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{Q^2/9}{2C} + \frac{4Q^2/9}{4C} = \frac{Q^2}{6C}$ [5 points]

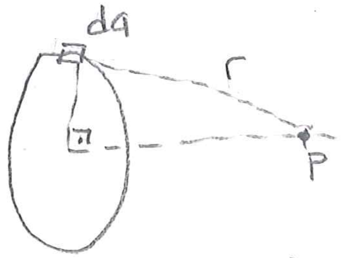
(c) $E_{lost} = E_{initial} - E_{final}$

$= \frac{Q^2}{2C} - \frac{Q^2}{6C} = \frac{2Q^2}{6C} = \frac{Q^2}{3C}$ [5 points]

(3)

3. Two parallel thin rings of radius R are located at a distance d from each other on the same axis. Find the work done by electric forces when moving charge q_0 from the center of the first ring to the center of the second. The first ring has the total charge of q_1. The second ring has the total charge of charge q_2.





Potential on the axis

$$\int dV = \int \frac{k\,dq}{r} = \frac{k}{r}\int dq =$$

$$V = \frac{kQ_{total}}{r}$$ 4 points

$$V_A = \sum \frac{kQ_i}{r_i} = \frac{kq_1}{R} + \frac{kq_2}{\sqrt{d^2+R^2}}$$ 4 points

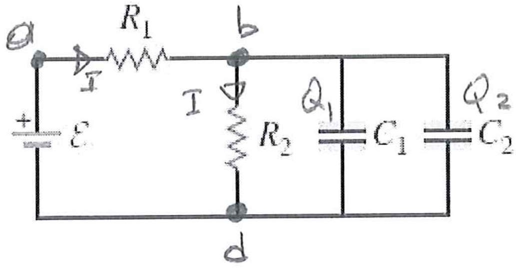
$$V_B = \sum \frac{kQ_i}{r_i} = \frac{kq_1}{\sqrt{d^2+R^2}} + \frac{kq_2}{R}$$ 4 points

$$V_A - V_B = k(q_1-q_2)\left(\frac{1}{R} - \frac{1}{\sqrt{d^2+R^2}}\right)$$ 4 points

$$W_{A\to B} = q_0(V_A - V_B) = kq_0(q_1-q_2)\left(\frac{1}{R} - \frac{1}{\sqrt{d^2+R^2}}\right)$$

4 points

5. Consider the circuit shown in the figure. The battery has emf $\mathcal{E} = 72.0$ V and negligible internal resistance. $R_2 = 2.00\ \Omega$, $C_1 = 3.00\ \mu F$, and $C_2 = 6.00\ \mu F$. After the capacitors have attained their final charges, the charge on C_1 is $Q_1 = 18.0\ \mu C$.
(a) What is the final charge on C_2.
(b) What is the resistance R_1?



(a) $V_{bd} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

10 points

$\frac{18\ \mu C}{3{,}0\ \mu F} = \frac{Q_2}{6{,}0\ \mu F}$ $\Rightarrow Q_2 = 36{,}0\ \mu C$

(b) $V_{bd} = \frac{18{,}0\ \mu C}{3{,}0\ \mu F} = 6$ volt $= I R_2$

$I = \frac{6\text{ volt}}{2\ \Omega} = 3{,}00$ A

5 points

$\mathcal{E} = V_{ad} = V_{ab} + V_{bd}$

$72 = I R_1 + 6$ volt

5 points

$R_1 = \frac{72-6}{I} = \frac{66\text{ volt}}{3\text{ A}} = 22{,}0\ \Omega$

6. **(Bonus, 2 points each)**

Determine if the statement is **true** or **false**. Circle your answer.

(a) If the electric flux through a closed surface is zero, the electric field at points on that surface must be zero.

A) True (B) False 2 points

(b) If the electric field is zero everywhere inside a region of space, the potential must also be zero in that region.

A) True (B) False 2 points

(c) When the electric field is zero at a point, the potential must also be zero there.

A) True (B) False 2 points

(d) If the electric potential at a point in space is zero, then the electric field at that point must also be zero.

A) True (B) False 2 points

(e) As current flows through a uniform wire, the wire gets hotter because the electrons stop moving and therefore transform their lost kinetic energy into thermal energy in the wire.

A) True (B) False 2 points