PHYS121 2021-2022 Fall Semester
Second Midterm

SOLUTIONS

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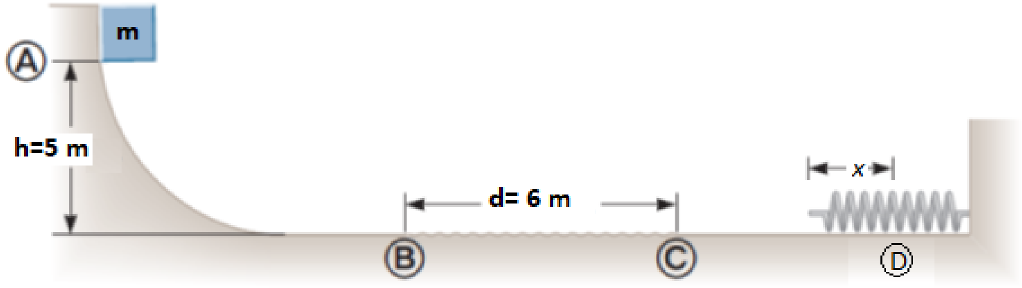
1)- A block of mass *m* released from point A reaches point D. The track is frictionless except for the portion between points B and C, which has a length of *d* . The coefficient of kinetic friction between the block and the rough surface between points B and C is μ_k. The block hits a spring of force constant *k*, and compresses the spring from its equilibrium position before coming to rest momentarily.
(m = 2 kg , h = 5 m, d = 6m, k = 320 N/m, g = 10 m/s^2, μ_k= 1/2)

a) What is the velocity of the block at point B?
b) Calculate the kinetic energy of the block at point C.
c) Calculate the amount of compression in the spring.



a) $mgh + 0 = \frac{1}{2} m v_B^2 + 0 \Rightarrow v_B = \sqrt{2gh} = \sqrt{2\cdot 10 \cdot 5}$

6 $v_B = 10$ m/s

b) $U_B + K_B + W_{fr} = K_C + U_C$ $W_{fr} = \vec{F}_{fr}\cdot\vec{d} = \mu \underbrace{N}_{mg} \cdot d \underbrace{\cos 180}_{-1}$

8 $0 + \frac{1}{2} m v_B^2 + \vec{f}_{sür}\cdot\vec{d} = K_C + 0$ $= -\mu m g d = -\frac{1}{2}\cdot 2 \cdot 10 \cdot 6$

$= -60$ Joule

$\frac{1}{2}\cdot 2 \cdot 100 - 60 = K_C$

$\boxed{K_C = 40 \text{ Joule}}$

c) 6 $K_C + \underbrace{\cancel{U_C}}_{0} + \underbrace{\cancel{W_f}}_{0} = \underbrace{\cancel{K_D}}_{0} + U_D \Rightarrow K_C = \frac{1}{2} k x^2$

$x = \sqrt{\frac{2\cdot K_C}{k}} = \sqrt{\frac{2\cdot 40}{320}} = \sqrt{\frac{1}{4}}$ $x = \frac{1}{2}$ m

2)- An object of mass 2.00 kg, moving with an initial velocity of $-5.00\,\hat{\imath}$ m/s, collides with and sticks to an object of mass 3.00 kg with an initial velocity of $3.00\,\hat{\jmath}$ m/s. Find the final velocity, magnitude of the velocity and direction of the velocity of the composite object.

$m_1 = 2.00$ kg

$\bar{v}_1 = -5.00\,\hat{i}$ (m/s)

$m_2 = 3.00$ kg

$\bar{v}_2 = 3.00\,\hat{j}$ (m/s)

$$\bar{P}_i = \bar{P}_f$$

$$m_1 \bar{v}_1 + m_2 \bar{v}_2 = (m_1 + m_2)\,\bar{v} \quad 2.5$$

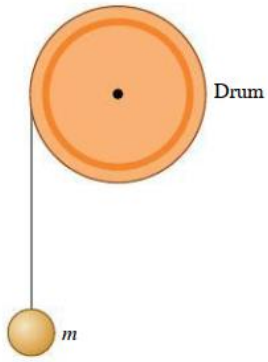
$$\Rightarrow \bar{v} = \frac{m_1 \bar{v}_1 + m_2 \bar{v}_2}{(m_1 + m_2)} \Rightarrow \bar{v} = \frac{-10\,\hat{i} + 9\,\hat{j}}{5}$$

$$\Rightarrow \boxed{\bar{v} = -2.0\,\hat{i} + 1.80\,\hat{j} \ \text{(m/s)}} \quad 7.5$$

$$|\bar{v}| = \sqrt{(-2)^2 + (1.8)^2} \Rightarrow \boxed{v = 2.69 \ \text{m/s}} \quad 5$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{1.8}{2} \Rightarrow \boxed{\theta \simeq 42^\circ} \quad 5$$

3)- A uniform drum in the shape of a solid disk is mounted on a frictionless axle at its center. The drum has mass 5.00 kg and radius 0.800 m. A thin rope is wrapped around the drum, and a block is suspended from the free end of the rope. The system is released from rest and the block moves downward. What is the mass of the block if the wheel turns through 8.00 revolutions in the first 5.00 s after the block is released? $I_{CM} = \frac{1}{2}MR^2$



Answer 2

$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

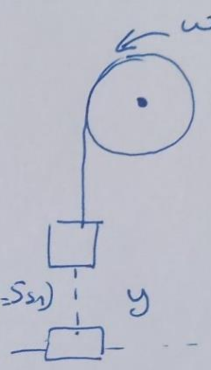
1 rev = 2π rad

8 rev = 16π rad

$16\pi \text{ rad} = 0 + \frac{1}{2}\alpha(5.0)^2$ (5)

$32\pi \text{ rad} = \alpha\, 25$

$\boxed{\alpha = 4{,}02 \text{ rad/s}^2}$



$\omega = \omega_0 + \alpha t$

$\omega = (4.02)(5.0) = 20.1 \text{ rad/s}$

$\boxed{\omega = 20{,}1 \text{ rad/s}}$ (5)

$U_1 + K_1 + W = U_2 + K_2$ (2,5)

$W = 0$

$U_2 = 0$

$U_1 = mgy$

$v = \omega R$

$y = ?$ $\boxed{R \cdot 16\pi \text{ rad} = y}$ (2,5)

$mgy = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgy = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}I\omega^2$

$mgy = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$

$y = 0{,}800\,(16\pi \text{ rad}) = 40.21 \text{ m}$

$mgy = \frac{1}{2}mR^2\omega^2 + \frac{1}{4}MR^2\omega^2$

$m\left(gy - \frac{1}{2}R^2\omega^2\right) = \frac{1}{4}MR^2\omega^2$ (5)

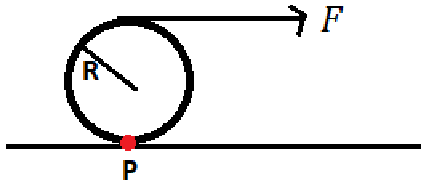
$$m = \frac{\frac{1}{4}MR^2\omega^2}{gy - \frac{1}{2}R^2\omega^2} = \frac{\frac{1}{4}(5)(0{,}8^2)(20{,}1)^2}{(9{,}8)(40{,}21) - \left(\frac{1}{2}(0{,}8^2)(20.1)^2\right)} = \boxed{1.22 \text{ kg}}$$

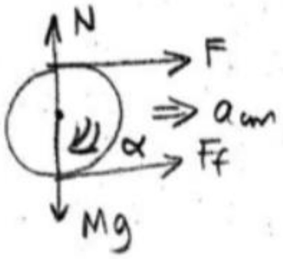
4)- A spool of wire of mass *M* and radius *R* is unwound under a constant horizontal force *F* as shown in the figure. Assuming the spool rolls without slipping:

a)- Find the acceleration of the center of the mass, a_{CM}.

b)- The angular acceleration, α , about the center of mass

c)- The linear acceleration of point P on the spool which is the contact point between the cylinder and the floor. ($I_{cylinder}=(MR^2)/2$)





$$F + F_f = M a_{cm}$$

$$F\cancel{R} - F_f \cancel{R} = \frac{MR^{\cancel{2}}}{2}\alpha$$

$$a_{cm} = R\alpha$$

$+$

$$2F = \frac{3M a_{cm}}{2}$$

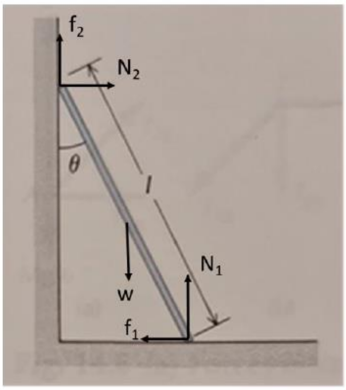
a) $\dfrac{4F}{3M} = a_{cm}$ //

b) $\dfrac{4F}{3MR} = \alpha$ //

c) $a_p = 0$ $\Leftarrow\Rightarrow$ $\dfrac{4F}{3M} = a_{cm}$

$R\alpha = \dfrac{4F}{3M}$

5)- The bottom end of a meter stick rests on the floor and the top end rest against wall. If the coefficient of static friction between **the stick** and **the floor** and **wall** is μ_s= 0.4, what is the maximum angle that the stick can make with the wall without slipping?



(4P)

$f_1 = \mu_s N_1 \qquad f_2 = \mu_s N_2$ (1P)

$lN_2 \cos\theta + l\mu_s N_2 \sin\theta - \frac{l}{2} Mg \sin\theta = 0$ (2P)

$-\mu_s N_1 + N_2 = 0$ (1P)

$\mu_s N_2 + N_1 - Mg = 0$ (1P)

$N_1 = \frac{Mg}{(\mu_s^2+1)}$ (1P)

$N_2 = \mu_s Mg/(\mu_s^2 + 1)$ (1P)

$\frac{\mu_s Mgl}{\mu_s^2+1} \cos\theta + \frac{\mu_s^2 Mgl}{\mu_s^2+1} \sin\theta - \frac{Mgl}{2} \sin\theta = 0$ (3P)

$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{2\mu_s}{1-\mu_s^2} = \frac{2\times0.4}{1-(0.4)^2} = 0.95$ (4P)

$\theta = 44°$ (2P)