PHYS121 2021-2022 Fall Semester
First Midterm
90 minutes

1	2	3	4	5	Total

Name Surname: ………………………………………. Student No: ....Lecturer:…………………………….....

Calculators are allowed but not their exchange. Each question is worth 20 points
Take g=9,80 m/s^2. **Good luck.**

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1. The two vectors are given by $\vec{a} = 3\hat{\imath} + 5\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath}$ in three dimensional cartesian coordinate system. Find
   a) the length of $2\vec{a} - 3\vec{b}$,
   b) $2\vec{a} \cdot 3\vec{b}$,
   c) the angle ϕ between the vectors $\vec{a}$ and $\vec{b}$, and
   d) the component of $\vec{a}$ along the direction of $\vec{b}$.

5pt a) $2\vec{a} = 6\hat{i} + 10\hat{j}$, $-3\vec{b} = -6\hat{i} - 12\hat{j}$ → $2\vec{a} - 3\vec{b} = -2\hat{j}$, $|2\vec{a} - 3\vec{b}| = 2$ //

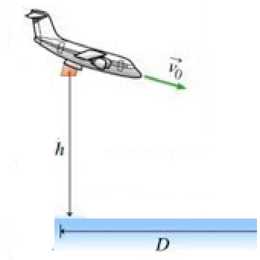
5pt b) $2\vec{a} = 6\hat{i} + 10\hat{j}$, $3\vec{b} = 6\hat{i} + 12\hat{j}$ → $2\vec{a} \cdot 3\vec{b} = 36 + 120 = 156$ //

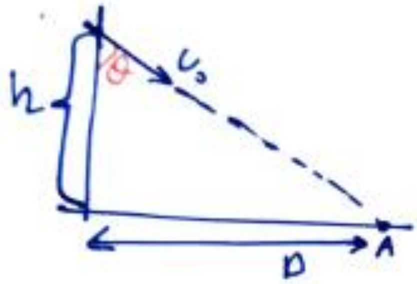
5pt c) $\vec{a} \cdot \vec{b} = ab\cos\phi$

$$\phi = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{ab}\right) = \cos^{-1}\left(\frac{26}{5.84 \times 4.47}\right) = \cos^{-1}(0.997) = 4.4^\circ$$

5pt d) $$\vec{a} \cdot \frac{\vec{b}}{b} = a\cos\phi = \frac{\vec{a}\cdot\vec{b}}{b} = \frac{26}{\sqrt{2^2+4^2}} = 5.81$$ //

2. A plane, diving with constant speed at an angle of 53.0° with the vertical, releases a projectile at an altitude of h=1000 m. The projectile hits the ground 4.0 s after release.
   a) What is the speed of the plane?
   b) How far does the projectile travel horizontally (D) during its flight?
   c) What are the horizontal and vertical components of its velocity just before striking the ground?





$\theta = 53°$
$h = 1000m$
$t_A = 4s.$

a) $t = t_A$ 'da $y = 0$

(10)

$y = y_0 - V_{0y} t - \frac{1}{2} g t^2$

$0 = 1000 - V_{0y} \cdot 4 - \frac{1}{2}(9,8)(4)^2$

$1000 = 4V_{0y} + \frac{1}{2}(9,8) \cdot 4^2 \Rightarrow V_{0y} = 250 - 2 \cdot 9,8$

250

$V_{0y} = 230,4 \, m/s$

$V_{0y} = V_0 \cos\theta_0 \Rightarrow V_0 = \frac{V_{0y}}{\cos\theta_0} = \frac{230,4}{0,6}$

$V_0 = 384 \, m/s$

b) (5)

$D = V_{0x} \cdot t = V_0 \sin 53 = 384 \cdot (0,8) \cdot 4$

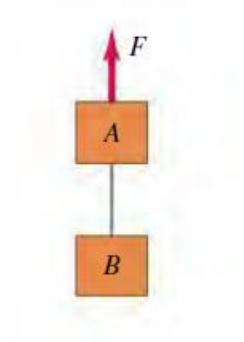
$D = 1228,8 \, m$

c) (5)

$V_x = V_{0x} = V_0 \sin 53 = 384 \cdot 0,8 = 307,2 \, m/s$

$V_y = -V_{0y} - gt = -230,4 - 9,8 \cdot 4 = -269,6 \, m/s$

3. Boxes A and B are connected to each end of a light vertical rope. A constant upward force F=80.0 N is applied to box A. Starting from rest, box B goes down 12.0 m in 4.00 s. The tension in the rope connecting the two boxes is 36.0 N. What are the masses of box B and box A?



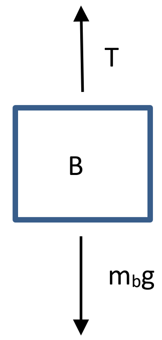
Solution of 3:

$v_0 = 0$

s=12m

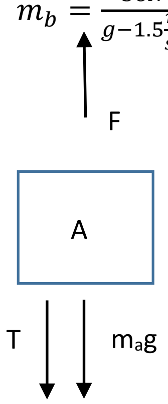
t=4s$s = v_0 t + \frac{1}{2}at^2 \quad s = \frac{1}{2}at^2$

$a = \frac{2s}{t^2} = \frac{2 \cdot 12m}{(4s)^2} = 1.5\frac{m}{s^2}$ (6 puan)



$m_b g - 36 = m_b 1.5\frac{m}{s^2}$

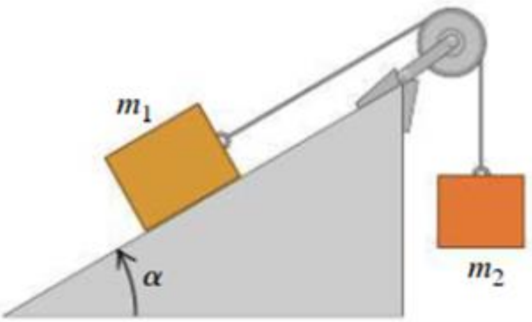
$m_b = \frac{36N}{g - 1.5\frac{m}{s^2}} = \frac{36N}{9.8\frac{m}{s^2} - 1.5\frac{m}{s^2}} = 4.337kg$ (7 Puan)



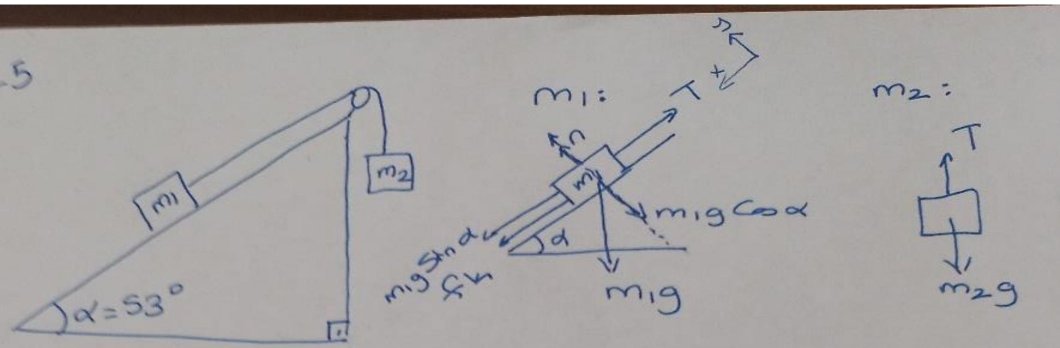
$(m_a g + T) - F = m_a a$

$m_a = \frac{F - T}{g - a} = \frac{80N - 36N}{9.8\frac{m}{s^2} - 1.5\frac{m}{s^2}} = 5.30kg$ (7 puan)

4. In the figure, block m_1 (20.0 kg) is placed on an inclined surface where α is 53º. The coefficient of kinetic friction (μ_k) between the block m_1 and the incline is 0.40. What must be the mass m_2 of the hanging block if it goes down 12.0 m in the first 3.0 s after the system is released from rest? Draw a free body diagram for each block and calculate the tension in the rope.



A-5



$y = y_0 + v_0 t + \frac{1}{2} a t^2$ ($v_0 = 0$)

$y - y_0 = \frac{1}{2} a t^2$

$a = \frac{2(y-y_0)}{t^2} = \frac{2 \cdot 12}{3^2} = 2,67\ m/s^2$

$\boxed{a = 2,67\ m/s^2}$

m_1:

$\Sigma F_y = m_1 a_y$ $\quad n = m_1 g \cos\alpha = (20)(9,8)\cos(53)$

$\boxed{n = 117.6\ N}$

$f_k = \mu_k n \Rightarrow f_k = (0,40)(117.6) = 47.04\ N$

$\boxed{f_k = 47.04\ N}$

$\Sigma F_x = m_1 a_x$

$T - f_k - m_1 g \sin 53 = m_1 a$

$T = f_k + m_1 g \sin 53 + m_1 a \Rightarrow T = 47.04 + 20 \cdot 9,8 \sin 53 + 20 \cdot 2,67$

$T = 47.04 + 156.8 + 53.4$

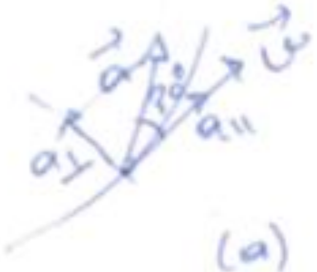
$\boxed{T = 257,24\ N}$

m_2:

$\Sigma F_y = m a_y$ $\quad m_2 g - T = m_2 a$ $\quad m_2 = \frac{T}{g-a} = \frac{257.24}{9.8 - 2,67}$

$\boxed{m_2 = \frac{257.24}{7.13} = 36.07\ kg}$

5. A particle revolves in a horizontal circle of radius 2.00 m. At a particular instant, its acceleration is $1.00\ m/s^2$ in a direction that makes an angle of 30.0° to its direction of motion. Assume that the magnitude of the tangential acceleration is constant. Determine its speed (*a*) at this moment, and (*b*) 1.00 s later.



$a_\perp = a \sin(30°) = 0.5\ m/s^2$

$a_\parallel = a \cos(30°) = 0.5\sqrt{3}\ m/s^2$

5 points

(a) $a_\perp = \frac{v^2}{R}$

$v^2 = R a_\perp = 2 \cdot 0.5 = 1$ 5 points

$v = 1.0\ m/s$ at $t=0$

(b) $\Delta v = a_\parallel \cdot \Delta t = 0.5\sqrt{3}\ m/s$ 5 points

$v(t=1.0s) = v(t=0.0s) + \Delta t$

$v = (1.0 + 0.5\sqrt{3})\ m/s$

5 points