PHYS121 2023-2024 Fall Semester
13 November 2023 First Midterm
90 minutes

1	2	3	4	5	Total

Name Surname: ………………………………………… Student No: ……………………………………….Lecturer:……………………………..........

Calculators are allowed but not their exchange. Each question is worth 20 points
Take g=9,80 m/s^2. **Good luck.**

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1. Vector $\vec{a}$ lies in the yz plane 63° from the positive direction of the y axis, has a positive z component, and has magnitude 3.20 units. Vector $\vec{b}$ lies in the xz plane 48° from the positive direction of the x axis, has a positive z component, and has magnitude 1.40 units. Find;
   (a) $\vec{a} \cdot \vec{b}$,
   (b) $\vec{a} \times \vec{b}$, and
   (c) the angle between $\vec{a}$ and $\vec{b}$

Q1

$\vec{a}$ on yz

$a_x = 0$
$a_y = 3.2\cos 63 = 1.45$
$a_z = 3.2 \sin 63 = 2.85$

$\vec{b}$ on xz

$b_x = 1.4 \cos 48 = 0.94$
$b_y = 0$
$b_z = 1.4 \sin 48 = 1.04$

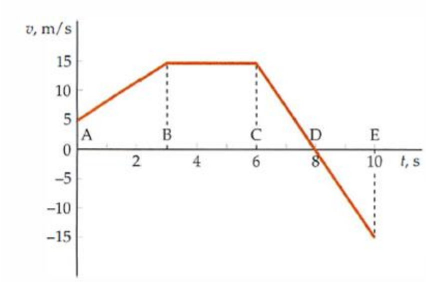
$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = 2.97$

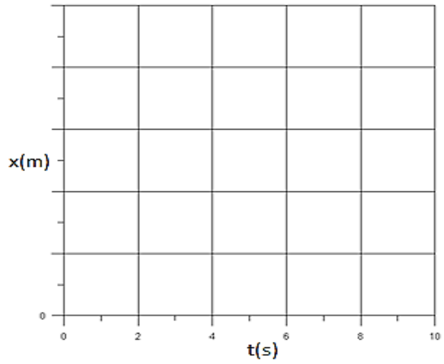
$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = 3.2$
$|\vec{b}| = 1.4$

$\theta = \cos^{-1}\left(\frac{2.97}{3.2 \times 1.4}\right)$
$\theta = 48°$

$\vec{a} \times \vec{b} = 1.5\hat{i} + 2.6\hat{j} - 1.36\hat{k}$

2. The one-dimensional motion of a particle is plotted in Figure.
   a) What is the average acceleration in the intervals: AB, BC, and CE?
   b) How far is the particle from its starting point after 10 s?
   c) Sketch the displacement of the particle as a function of time; label the instants A, B, C, D, and E on your figure.
   d) At what time is the particle traveling most slowly?





We can use the definition of average acceleration ($a_{av} = \Delta v/\Delta t$) to find a_{av} for the three intervals of constant acceleration shown on the graph.

(*a*) Using the definition of average acceleration, find a_{av} for the interval AB:

$$a_{av,AB} = \frac{15\,\text{m/s} - 5\,\text{m/s}}{3\,\text{s}} = \boxed{3.33\,\text{m/s}^2}$$

Find a_{av} for the interval BC:

$$a_{av,BC} = \frac{15\,\text{m/s} - 15\,\text{m/s}}{3\,\text{s}} = \boxed{0}$$

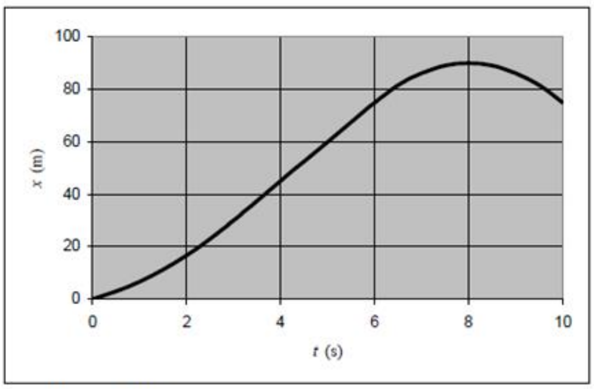
Find a_{av} for the interval CE:

$$a_{av,CE} = \frac{-15\,\text{m/s} - 15\,\text{m/s}}{4\,\text{s}} = \boxed{-7.50\,\text{m/s}^2}$$

(*b*) Use the formulas for the areas of trapezoids and triangles to find the area under the graph of *v* as a function of *t*.

$$\begin{aligned}\Delta x &= (\Delta x)_{A\to B} + (\Delta x)_{B\to C} + (\Delta x)_{C\to D} + (\Delta x)_{D\to E} \\ &= \tfrac{1}{2}(5\,\text{m/s} + 15\,\text{m/s})(3\,\text{s}) + (15\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}(15\,\text{m/s})(2\,\text{s}) + \tfrac{1}{2}(-15\,\text{m/s})(2\,\text{s}) \\ &= \boxed{75.0\,\text{m}}\end{aligned}$$

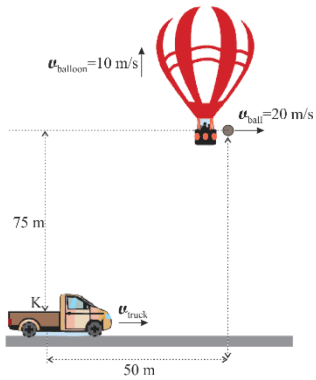
(*c*) The graph of displacement, *x*, as a function of time, *t*, is shown in the following figure. In the region from B to C the velocity is constant so the *x*- versus-*t* curve is a straight line.



(*d*) Reading directly from the figure, we can find the time when the particle is moving the slowest.

At point D, $t = 8$ s, the graph crosses the time axis; therefore, $v = 0$.

3. A hot air balloon is ascending with a constant speed of 10 m/s in air (where air resistance is negligible). A passenger in the balloon has noticed a truck moving with a constant speed $\boldsymbol{v}_{\text{truck}}$ relative to the ground when the passenger is at the position shown in the figure. The passenger has thrown a ball horizontally with a speed of 20 m/s in the direction of the truck's motion relative to the balloon. If the ball lands at point K on the truck, what is the magnitude of the truck's speed in m/s?



when the ~~balloon~~ ball reach the max. height

$v_y = v_{oy} - g t_1 = 0$, t_1 = time to reach the max. height

$t_1 = \frac{v_{oy}}{g} = \frac{10\,m/s}{9.8\,m/s^2} = 1.0s$

the time required for the ~~balloon~~ ball to come back to the initial height is $2t_1 = 2s$

Now the ~~balloon~~ ball has an initial velocity in $-y$ direction. So,

$y_f - y_i = v_{oy} t_2 - \frac{1}{2} g t_2^2$

$(0 - 75m) = (-10\,m/s)\,t_2 - \frac{1}{2}(9.8\,m/s^2)\,t_2^2$

$4.9t_2^2 + 10t_2 - 75 = 0$

$$\left\{\begin{array}{l} \text{recall} \\ ax^2 + bx + c = 0 \\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{array}\right\}$$

$t_2 = \frac{-10 \pm \sqrt{100 - (19.6)(-75)}}{9.8}$

$t_2 = 3.0s$

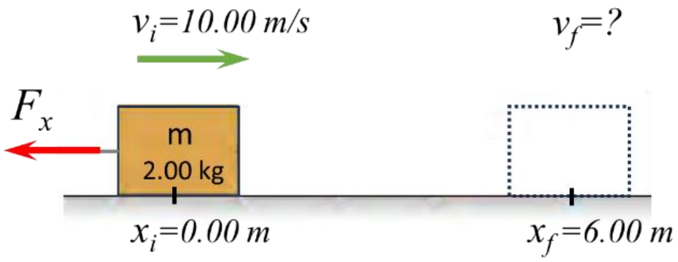
total time for the ~~balloon~~ ball to move in air is $t_{total} = 2t_1 + t_2 = 2.0s + 3.0s$
$= 5.0s$

~~Balloon~~ Ball takes $\Delta x = v_{ox} t = (20\,m/s)(5s) = 100m$ in horizontal direction in 5s

Then the truck has to take 150m in total in 5s.

so, $\Delta x = v_{truck} t_{total} \Rightarrow v_{truck} = \frac{\Delta x}{t_{total}} = \frac{150m}{5s} = 30\,m/s$

4. A net force along the x-axis that has x-component $F_x = -12\ \mathrm{N} + (0.500\ \mathrm{N/m^2})x^2$ is applied to a 2.00-kg object that is initially at the origin and moving in the x-direction with a speed of 10.00 m/s. What is the speed of the object when it reaches the point x = 6.00 m?



Solution

The force does work on the object, which changes its kinetic energy, so the work-energy theorem applies.

Note: The force is variable so we must integrate to calculate the work it does on the object

(work-energy theorem) 3 puan

$$W_{total} = \Delta K = K_f - K_i$$

(writing integration) 3 puan

(writing kinetic energy) 3 puan

$$\int_{x_1}^{x_2} F(x)\,dx = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\int_{x_i=0}^{x_f=6} \left(-12 + (0,5)x^2\right) dx = \frac{1}{\cancel{2}} \cdot \cancel{2}\, v_f^2 - \frac{1}{\cancel{2}} \cdot \cancel{2}\,(10)^2$$

$$\left[-12x + 0.5\frac{x^3}{3}\right]_{x_i=0m}^{x_f=6m} = v_f^2 - 100\left(\mathrm{kg}\frac{m^2}{s^2}\right)$$

$$(-12)(6) + \frac{(0,5)(6)^3}{3} = v_f^2 - 100$$

$$-72 + 36 = v_f^2 - 100$$

$$\Rightarrow v_f^2 = 64\ \frac{m^2}{s^2}$$

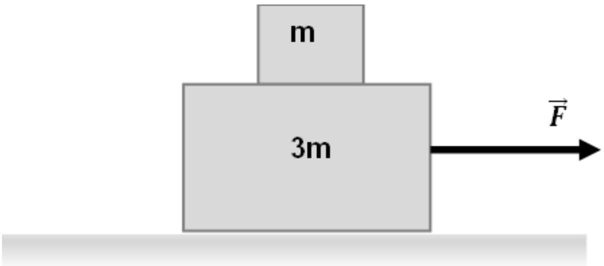
At least at the end (do not forget to use unit!)

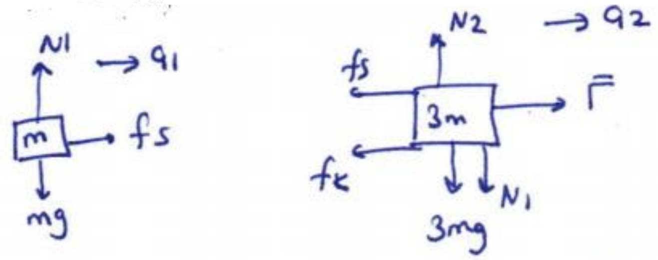
birim : 1 puan

result : 10puan

$$\boxed{v_f = 8.00\ m/s}$$

5. Consider two stacked blocks one on top of the other. The bottom block has mass 3m and rests on a horizontal floor. The top block has mass m. Suppose the coefficient of static and kinetic friction between all the surfaces are μ_s and μ_k, respectively. Find the maximum value of the magnitude of force $\vec{F}$ with which the lower block can be pulled horizontally so that the two blocks move together without slipping?





Blokk m: $f_s = m a_1$

$N_1 - mg = 0$

Block 3m:

$F - f_s - f_k = 3m a_2$

$N_2 - N_1 - 3mg = 0$

Since blocks move together, they have same acceleration

$a_1 = a_2 = a$

$f_s \le \mu_s N_1$ $\quad$ $f_k = \mu_k N_2 = \mu_k (N_1 + 3mg) = \mu_k (mg + 3mg)$

$= 4 \mu_k mg$

$F - m_1 a - 4\mu_k mg = 3m a \Rightarrow a = \dfrac{F - 4\mu_k mg}{4m}$

$m a = f_s \le \mu_s N_1$

$m\left(\dfrac{F - 4\mu_k mg}{4m}\right) \le \mu_s mg$ $\quad$ $F \le 4\mu_s mg + 4\mu_k mg$

$F_{max} = 4mg(\mu_s + \mu_k)$