PHYS121 2023-2024 Fall Semester
2nd Midterm Time: 90 minutes

1	2	3	4	5	Toplam

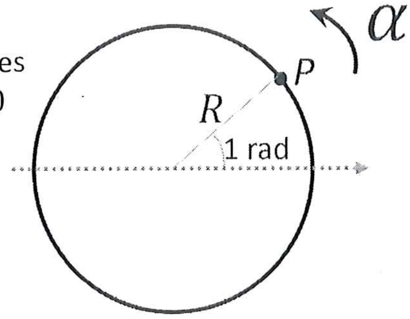
Student No:...

Name Surname:.. Lecturer:..

You can use calculator during the exam, but exchanging is not allowed.

Unless stated otherwise, take $g = 9.80$ m/s² if necessary. Each question worth 20 points. **Good luck**

1. A wheel $D = 2R = 2.00$ m in diameter lies in a vertical plane and rotates about its central axis with a constant angular acceleration of $\alpha = 4.00$ rad/s². The wheel starts at rest at $t = 0$, and the radius vector of a certain point P on the rim makes an angle of 1.00 radian with the horizontal at $t = 0$.



a) Find the angular speed of the wheel at $t = 2.00$ s.

$\omega_f = \omega_i + \alpha t = 0 + 4.00\,\frac{rad}{s^2} \cdot 2{,}00s = 8{,}00\,\frac{rad}{s}$ 3 puan

b) Find the tangential speed of point P at $t = 2.00$ s.

$v = R\omega = 1{,}00\,m \cdot 8{,}00\,\frac{rad}{s} = 8{,}00\,m/s$ 3 puan

c) Find the radial and tangential components of the total acceleration of point P at $t = 2.00$ s. What is the magnitude of the total acceleration?

$a_r = \frac{v^2}{R} = \frac{(8{,}00)^2\,m^2/s^2}{1{,}00\,m} = 64{,}0\,m/s^2$ 3 puan

$a_t = R\alpha = 1{,}00\,m \cdot 4{,}00\,rad/s^2 = 4{,}00\,m/s^2$ 3 puan

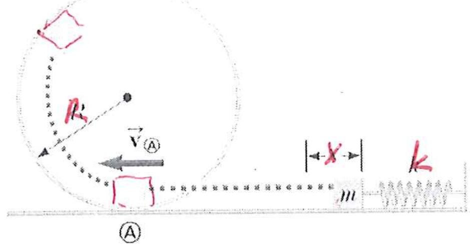
$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64{,}0)^2 + (4{,}00)^2}\,m/s^2 = 64{,}1\,m/s^2$ 3 puan

d) Find the angular position of point P in radians at $t = 2.00$ s.

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = 1{,}00\,rad + 0 \cdot 2 + \frac{1}{\cancel{2}}\,\overset{2}{\cancel{4{,}00}}\,\frac{rad}{s^2}(2{,}00\,s)^2$ 3 puan

$= 1{,}00\,rad + 8{,}00\,rad = 9{,}00\,rad$ 2 puan

2. A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance x (see the figure). The force constant of the spring is $k = 200$ N/m. When it is released, the block travels along a frictionless, horizontal surface to point A, the bottom of a vertical circular track of radius $R = 1.00$ m and continues to move up the track. The block's speed at the bottom of the track is $v_A = 12.0$ m/s, and the block experiences an average friction force of 6.00 N while sliding up the track.



(a) What is x?

$\frac{1}{2}kx^2 = \frac{1}{2}mv_A^2$

$x = \sqrt{\frac{m}{k}}\,v_A = \sqrt{\frac{0{,}500}{200}} \cdot 12{,}0 = 0{,}600$ m

4 puan

(b) If the block were to reach the top of the track, what must be the minimum speed at that point?

mg

n

at least

$\cancel{m}g = \cancel{m}\frac{v^2}{R}$

$v = \sqrt{gR} = \sqrt{9{,}80 \cdot 1} = 3{,}13$ m/s

4puan

(c) If the block reaches the top, how much mechanical energy is converted into internal energy because of the friction?

$\Delta E_{inter} = f_k d = f_k \cdot \pi R = 6{,}00\,N \cdot \pi \cdot 1{,}00\,m = 6\pi$ J

$\Delta E_{inter} = 18{,}85$ J

4 puan

(d) What must be the minimum kinetic energy at A for the block to reach the top?

$\Delta K + \Delta U_{grav} + \Delta E_{inter} = 0$

$\frac{1}{2}mv^2 - K_A + mg\,2R + 18{,}85 = 0$

$K_A = \frac{1}{2}mv^2 + mg\,2R + 6\pi = \frac{1}{2} \cdot 0{,}5 \cdot 9{,}80 + 0{,}5 \times 9{,}80 \times 2 \cdot 1 + 18{,}85$

4 puan

$K_A = 31{,}1$ J

This is the minimum!

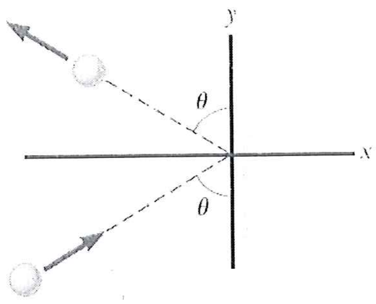
(e) Does the block actually reach the top of the track, or does it fall off before reaching the top?

Yes. The kinetic energy at A $\frac{1}{2}mv_A^2 = \frac{1}{2} \cdot 0{,}5 \cdot (12{,}0\,m/s)^2 = 36$ J

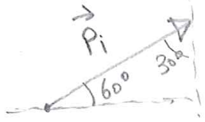
It has enough energy to reach the top.

4 puan

3. A 4.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of $\theta = 30.0°$ with the surface. It bounces off with the same speed and angle, as shown in the figure.



(a) What are the x and y components of the initial and final momentums of the ball?



$P_{ix} = 4.00\,kg \cdot 10.0\,m/s \cdot \cos(60°) = 20.0\ kg\,m/s$

$P_{iy} = 4.00\,kg \cdot 10.0\,m/s\ \sin(60°) = 34.6\ kg\,m/s$ $\boxed{4 \text{ puan}}$

$\vec{P}_i = (20.0\,\hat{i} + 34.6\,\hat{j})\ kg\,m/s$

$P_{fx} = -4.00 \cdot 10.0 \cdot \cos 60°\ \frac{kg\,m}{s} = -20.0\ kg\,m/s$

$P_{fy} = 4.00\,kg \cdot 10.0\,\frac{m}{s} \cdot \sin 60° = 34.6\ kg\,m/s$ $\boxed{4 \text{ puan}}$

$\vec{P}_f = (-20\,\hat{i} + 34.6\,\hat{j})\ kg\,m/s$

(b) What is the magnitude and direction of the momentum change?

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = (-20\,\hat{i} + 34.6\,\hat{j}) - (20\,\hat{i} + 34.6\,\hat{j}) = (-40.0\,\hat{i})\ kg\,m/s$

$-x$-direction $40.0\ kg\,m/s$ $\boxed{4 \text{ puan}}$

(c) What is the magnitude and direction of the impulse on the ball exerted by the wall?

$\vec{J} = \Delta\vec{P}$; $-x$ direction $40.0\ Ns$ $\boxed{4 \text{ puan}}$

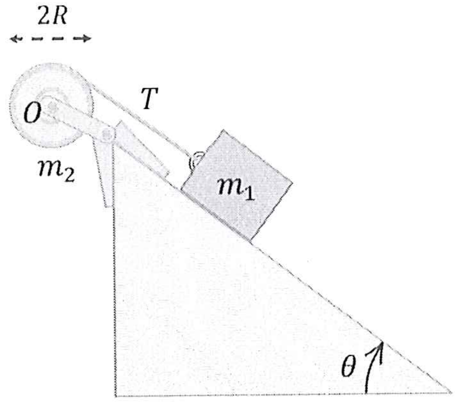
(d) If the ball is in contact with the wall for 0.200 s, what is the magnitude and direction of the average force exerted by the wall on the ball?

$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} \Rightarrow |\vec{F}_{avg}| = \frac{|\vec{J}|}{\Delta t} = \frac{40.0\,N\cancel{s}}{0.200\,\cancel{s}} = 200\,N$

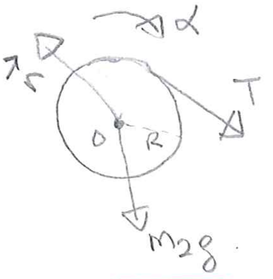
$\boxed{4 \text{ puan}}$

200 N in $-x$ direction

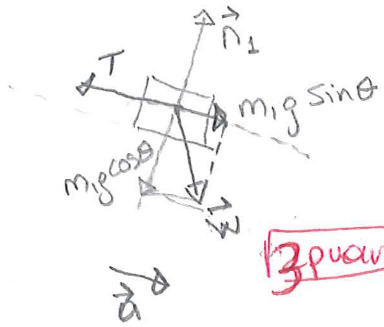
4. A block with mass m_1 starts from rest and slides down a frictionless surface inclined at an angle θ to the horizontal. A string attached to the block is wrapped around a pulley of mass m_2 and radius R on a fixed axis at O. The pulley is free to rotate (without friction) about a fixed axis through its center with a stationary rotation axle. Moment of inertia of the pulley is given by $I = \frac{1}{2} m_2 R^2$. The string is parallel to the inclined surface.



(a) Draw free-body diagrams for the pulley and box.



3 puan



3 puan

$|\vec{a}| = a$

2 puan

$\boxed{a = R\alpha}$

rolling without slipping

(b) Find the magnitude of the acceleration of the block in terms of m_1, m_2, g, and θ.

$TR = I\alpha$

$TR = \frac{1}{2} m_2 R^2 \alpha$

$T = \frac{m_2 R \alpha}{2}$

$T = \frac{m_2 a}{2}$

3 puan

$m_1 g \sin\theta - T = m_1 a$ 3 puan

$m_1 g \sin\theta - \frac{m_2 a}{2} = m_1 a$

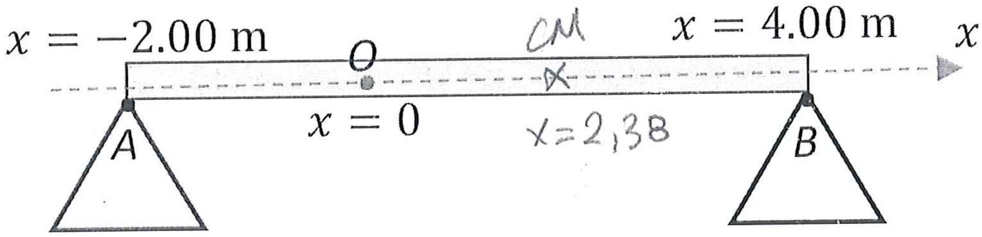
$a\left(m_1 + \frac{m_2}{2}\right) = m_1 g \sin\theta$

$a = \frac{m_1 g \sin\theta}{m_1 + \frac{m_2}{2}}$ 3 puan

(c)What is the tension T in the string in terms of m_1, m_2, g, and θ?

$T = \frac{m_2 a}{2} = \frac{m_1 m_2 g \sin\theta}{2m_1 + m_2}$ 3 puan

5. A thin non-uniform beam extends from point A ($x = -2.00$ m) to point B ($x = 4.00$ m), and rests horizontally on two triangles, as shown in the figure. The linear mass density of the beam is given by $\lambda = \frac{dm}{dx} = \left(0.100 \frac{\text{kg}}{\text{m}}\right) + \left(0.300 \frac{\text{kg}}{\text{m}^3}\right) x^2$.



(a) Find the total mass of the beam in kilograms.

$$M = \int dm = \int_{-2}^{4} (0{,}1 + 0{,}3x^2)\,dx = \left[0{,}1x + 0{,}1x^3\right]_{-2}^{4}$$

$$= (0{,}4 + 6{,}4) - (-0{,}2 - 0{,}8) \text{ kg}$$

$\boxed{4 \text{ points}}$

$$M = 7{,}8 \text{ kg}$$

(b) Find the center of mass coordinate x_{CM} of the beam.

$$x_{CM} = \frac{1}{M}\int x\,dm = \frac{1}{7{,}8}\int_{-2}^{4}(0{,}1x + 0{,}3x^3)\,dx = \frac{1}{7{,}8}\left(0{,}1\frac{x^2}{2} + 0{,}3\frac{x^4}{4}\right)_{-2}^{4}$$

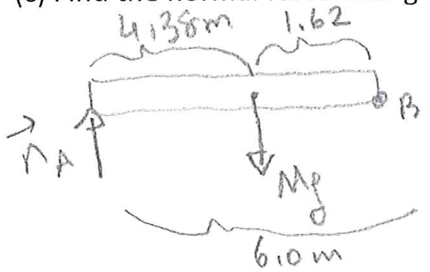
$\boxed{4 \text{ points}}$

$$= \frac{1}{7{,}8}\left(0{,}1 \cdot \frac{\overset{8}{\cancel{16}}}{\cancel{2}} + 0{,}3\frac{\overset{64}{\cancel{256}}}{\cancel{4}} - 0{,}1\frac{\overset{2}{\cancel{4}}}{\cancel{2}} - 0{,}3\frac{\overset{4}{\cancel{16}}}{\cancel{4}}\right)$$

$$x_{CM} = \frac{1}{7{,}8}(0{,}1 \times 6 + 0{,}3 \times 60) = \frac{18{,}6}{7{,}8} = 2{,}38 \text{ m}$$

$\boxed{4 \text{ points}}$

(c) Find the normal force acting on the beam at point A. Take torque with respect to B

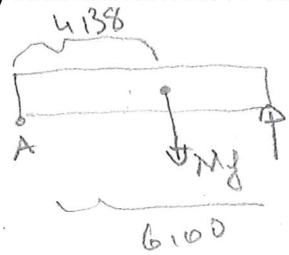


$\boxed{4 \text{ points}}$

$$n_A\, 6{,}0 = Mg\, 1.62$$

$$n_A = Mg\,\frac{1.62}{6{,}0} = 7{,}8 \times 9{,}8 \cdot \frac{1{,}62}{6{,}0} = 20.6 \text{ N}$$

(d) Find the normal force acting on the beam at point B. Take torque with respect to A



$\boxed{4 \text{ points}}$

$$n_B\, 6{,}00 = Mg\, 4{,}38$$

$$n_B = Mg\,\frac{4{,}38}{6{,}00} = 7{,}8 \times 9{,}8 \times \frac{4{,}38}{6{,}00} = 55{,}8 \text{ N}$$

alternatif or $n_B = Mg - n_A = 7{,}8 \times 9{,}80 - 20.6 = 55{,}8$ N