

1	2	3	4	5	Toplam

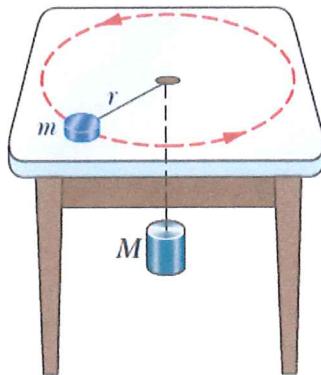
Student No:.....

Name Surname:..... Lecturer:.....

You can use calculator during the exam, but exchanging is not allowed.

Unless stated otherwise, take  $g = 9.80 \text{ m/s}^2$  if necessary. Each question worth 20 points. Good luck

1. A puck of mass  $m = 1.50 \text{ kg}$  slides in a circle of radius  $r = 20.0 \text{ cm}$  on a frictionless table while attached to a hanging cylinder of mass  $M = 2.50 \text{ kg}$  by means of a cord that extends through a hole in the table. What speed keeps the cylinder at rest?



The cylinder is at rest

if  $Mg = m \frac{v^2}{r}$

15 puan

$$v^2 = \frac{Mgr}{m}$$

$$r = 20 \text{ cm} = 0.2 \text{ m}$$

15 puan

Yiğit doğru olmustur

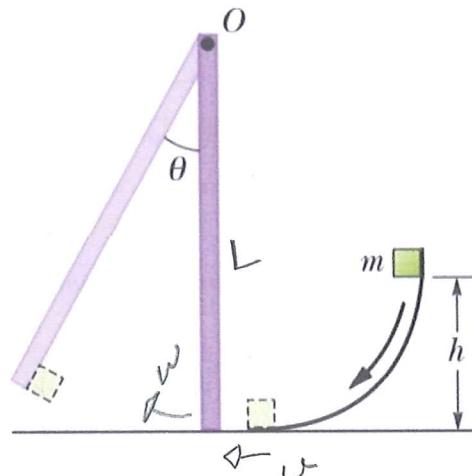
$$v = \sqrt{\frac{Mgr}{m}}$$

15 puan

$$v = \sqrt{\frac{2.50 \times 9.80 \times 0.2}{1.50}} = 1.81 \frac{\text{m}}{\text{s}}$$

5 puan  
Nümerik sonucu

2. In the figure, a small 50 g block slides down a frictionless surface through height  $h = 20 \text{ cm}$  and then sticks to a uniform rod of mass 100 g and length 40 cm. The rod pivots about point  $O$  and is initially at rest. With what angular speed will the rod start its rotation? You may treat the block as a point particle. (The moment of inertia about the end of a rod is given by  $I = \frac{1}{3}ML^2$ , where  $M$  is the mass, and  $L$  the length of the rod.)



$$m = 50 \text{ g} = 50 \times 10^{-3} \text{ kg}$$

$$M = 100 \text{ g} = 0,100 \text{ kg}$$

$$L = 40 \text{ cm} = 0,40 \text{ m}$$

$$h = 20 \text{ cm} = 0,20 \text{ m}$$

No friction, mechanical energy conserved

$$\Delta U + \Delta K = 0$$

$$(0 - mgh) + \frac{1}{2} mu^2 = 0$$

$$u = \sqrt{2gh}$$

5 puan

mechanical  
energy's  
conservation

speed just before  
the collision

During the collision, angular momentum conserved

$$I_{\text{total}} = mL^2 + \frac{1}{3}ML^2$$

Elemental  
momentum

$$L_f = L_{\text{initial}}$$

5 puan

$$Iw = muL$$

angular momentum  
conservation

$$w = \frac{muL}{I} = \frac{m \sqrt{2gh} L}{mL^2 + \frac{1}{3}ML^2} = \frac{\sqrt{2gh} L}{L + \frac{1}{3} \frac{M}{m} L} = \frac{\sqrt{2gh}}{L \left( 1 + \frac{1}{3} \frac{M}{m} \right)}$$

$$w = \frac{\sqrt{2 \times 9,8 \times 0,2}}{0,4 \left( 1 + \frac{1}{3} \cdot \frac{2,100}{50} \right)} = \frac{3 \sqrt{0,4 \times 9,8}}{2} \frac{\text{rad}}{\text{s}}$$

5 puan

cümle  
numere some  
bulmak

$$w = 3,0 \frac{\text{rad}}{\text{s}} \quad (2 \text{ significant figures})$$

3. An object is undergoing a simple harmonic motion with period 0.300 s and amplitude 6.00 cm. At  $t = 0$  the object is instantaneously at rest at  $x = 6.00$  cm. Calculate the (a) motion's frequency  $f$ , and (b) angular frequency  $\omega$ . (c) Find the time it takes the object to go from  $x = 6.00$  cm to  $x = -1.50$  cm.

$$T = 0.300 \text{ s}$$

$$A = 6.00 \text{ cm}$$

$$x(t) = A \cos(\omega t), \quad t=0, \quad x=A=6.0 \text{ cm}$$

$$v = \frac{dx}{dt} \Big|_{t=0} = -\omega A \sin(0) = 0$$

initial phase is "0"

$$(a) f = \frac{1}{T} = \frac{1}{0.3} \text{ s}^{-1} = 3.33 \text{ s}^{-1} \quad \boxed{4 \text{ puan}}$$

$$(b) \omega = \frac{2\pi}{T} = \frac{2\pi}{0.3} \text{ rad/s} = \frac{20\pi}{3} \text{ rad/s} = 20.9 \text{ rad/s} \quad \boxed{4 \text{ puan}}$$

$$(c) x(t) = (6.00 \text{ cm}) \cos\left(\frac{20\pi}{3}t\right) \quad \boxed{4 \text{ puan}}$$

Faz açısının  
"0°" olduğunu  
fark etmeli

$$x(t_i) = 6.00 \text{ cm} = (6.00 \text{ cm}) \cos\left(\frac{20\pi}{3}t_i\right) \Rightarrow t_i = 0$$

$$x(t_f) = -1.50 \text{ cm} = (6.00 \text{ cm}) \cos\left(\frac{20\pi}{3}t_f\right) \quad \boxed{4 \text{ puan}}$$

$$\cos\left(\frac{20\pi}{3}t_f\right) = -\frac{1}{4}$$

$$t_f = \frac{3}{20\pi} \cos^{-1}\left(-\frac{1}{4}\right) = 0.0871 \text{ s}$$

$\boxed{4 \text{ puan}}$

$$\Delta t = t_f - t_i = 0.0871 \text{ s}$$

4. A transverse harmonic wave travels on a rope according to the following expression:  
 $y(x, t) = 0.2 \sin(2.4x + 18t)$ , where the distances are in meters, and the time is in seconds.  
The mass density of the rope is  $\mu = 0.1 \text{ kg/m}$

- a) What is the travelling direction of the wave?
- b) What is the amplitude of the wave?
- c) What is the period of the wave?
- d) What is the wavelength of the wave?
- e) What is the tension on the rope?
- f) At  $x = 0.3 \text{ m}$  and  $t = 0.5 \text{ s}$ , what are the velocity and acceleration of the rope?

General formulae

$$y(x, t) = A \sin(kx + \omega t) \quad \begin{matrix} \text{wave in } -x\text{-direction} \\ \text{wave traveling in } +x\text{-direction} \end{matrix}$$

$$y(x, t) = A \sin(kx - \omega t)$$

(a)  $-x$ -direction. 2 puan

(b)  $A = 0.2 \text{ m}$  2 puan

(c)  $\omega = 18 \frac{\text{rad}}{\text{s}} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{18} \text{ s} = \frac{\pi}{9} \text{ s}$  2 puan

(d)  $k = 2.4 \text{ m}^{-1} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{2.4} = \frac{\pi}{1.2} \text{ m} = \frac{5\pi}{6} \text{ m}$  2 puan

(e)  $V = \frac{\omega}{k} = \sqrt{\frac{F}{\mu}} \Rightarrow F = \mu \cdot \left(\frac{\omega}{k}\right)^2$  2 puan

$$F = 0.1 \frac{\text{kg}}{\text{m}} \cdot \left(\frac{18 \text{ rad}}{\text{s}}\right)^2 = 5.625 \text{ kg} \frac{\text{m}}{\text{s}^2} = 5.625 \text{ N}$$

(f)  $\frac{dy}{dt} = \frac{\partial y}{\partial t} = 3.6 \cos(2.4x + 18t)$  2 puan  
 $= 3.6 \cos(2.4 \cdot 0.3 + 18 \cdot 0.5) = -3.44 \text{ m/s}$

$$\alpha = \frac{\partial V_y}{\partial t} = -64.8 \sin(2.4x + 18t)$$
2 puan

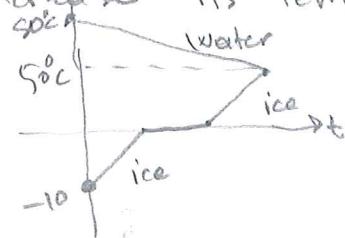
$$= -64.8 \sin(2.4 \cdot 0.3 + 18 \cdot 0.5)$$

$$= 18.85 \text{ m/s}^2$$

5. A thermally insulated container with negligible mass contains 0.50 kg of water at a temperature of  $80.0^{\circ}\text{C}$ . How many kilograms of ice cubes at a temperature of  $-10.0^{\circ}\text{C}$  must be put into the water to make the final temperature of the system  $50.0^{\circ}\text{C}$ ?

(The specific heat of water  $c_{\text{water}} = 4190 \text{ J/kg}\cdot\text{K}$ , the specific heat of ice  $c_{\text{ice}} = 2100 \text{ J/kg}\cdot\text{K}$ , and the latent heat of fusion of water  $L_f = 334 \times 10^3 \text{ J/kg}$ .)

ice must heat up to  $0^{\circ}\text{C}$  then melts. Finally it should increase its temperature to  $50^{\circ}\text{C}$



$$\sum \Delta Q = 0 \quad \boxed{3 \text{ puan}}$$

$$m_{\text{ice}} \cdot c_{\text{ice}} \cdot (0 - (-10)) + m_{\text{ice}} \cdot L_f \quad \boxed{3 \text{ puan}}$$

$$+ m_{\text{ice}} \cdot c_{\text{water}} \cdot (50 - 0) + m_{\text{water}} \cdot c_{\text{water}} \cdot (50 - 80) = 0 \quad \boxed{3 \text{ puan}}$$

$$m_{\text{ice}} = \frac{m_{\text{water}} \cdot 30 \cdot c_{\text{water}}}{10 \cdot c_{\text{ice}} + L_f + 50 \cdot c_{\text{water}}} \quad \boxed{3 \text{ puan}}$$

$$m_{\text{ice}} = \frac{0,50 \cdot 30 \cdot 4190}{10 \cdot 2100 + 334 \times 10^3 + 50 \cdot 4190} \text{ kg}$$

$$m_{\text{ice}} = 0,111 \text{ kg} \quad \boxed{2 \text{ puan}}$$

nümerik sonuc

Denklemdeki her terim 3'er puan