Experiment No : EM8

Experiment Name: Magnetic induction

Objective: Investigation of magnetic induction voltage as a function of frequency and magnitude of the magnetic field. Investigation of the induced voltage as a function of the cross-sectional area of the induction coil, frequency of current passing through the solenoid, and the number of windings.

Keywords: Magnetic force, magnetic flux, Faraday's law of induction,

Theoretical Information:

In 1831, Michael Faraday discovered that an electric current is produced in a closed conducting loop when the flux of magnetic field through the surface enclosed by this loop changes. This phenomenon is called *electromagnetic induction*, and the current produced an *induced current*.

The phenomenon of electromagnetic induction shows that when the magnetic flux in a loop changes, an induced electromotive force $\mathcal{E}_i$ is set up. The value of $\mathcal{E}_i$ does not depend on how the magnetic flux Φ is changed and is determined only by the rate of change of Φ, i.e. by the value of $d\Phi/dt$.

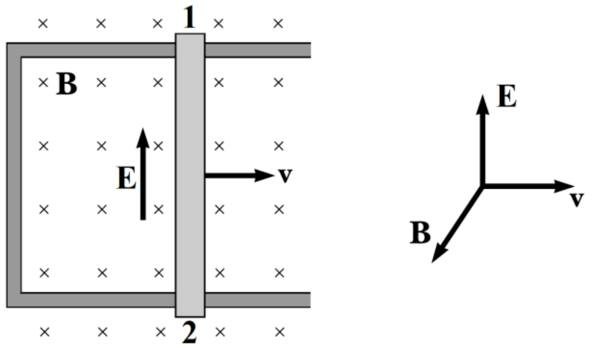

Figure 8.1: A conducting bar of length *l* on two fixed conducting rails is given an initial velocity **v** to the right.

A familiar example in Figure 8.1 to observe the magnetic induction is a loop with a movable rod of length *l*. If we put it in a homogeneous magnetic field at right angles to the plane of the loop and directed beyond the drawing, and bring the rod into motion with constant velocity **v**, the current carriers in the rod, i.e. electrons, will also begin to move relative to the field with the same velocity. As a result, each electron will begin to experience the Lorentz force

$$\mathbf{F}_i = -e\mathbf{v} \times \mathbf{B} \qquad 8.1$$

directed along the rod where the charge of an electron is $-e$. The action of this force is equivalent to the action on an electron of an electric field of strength,

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} \qquad 8.2$$

which is not electrostatic in origin. From the relation between electric field and electric potential, path integral of the induced field over the loop gives the induced electromotive force,

$$\mathcal{E}_i = \oint \mathbf{E}\, d\mathbf{l} = \oint \mathbf{v} \times \mathbf{B}\, d\mathbf{l} = \int_1^2 \mathbf{v} \times \mathbf{B}\, d\mathbf{l} = \mathbf{v} \times \mathbf{B} \int_1^2 d\mathbf{l} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad 8.3$$

where limits 1 and 2 indicate two tips of the movable rod. Here third equality holds, since the electric field is non-zero only on the movable part; the fourth one comes from the constancy of two vectors $\mathbf{v}, \mathbf{B}$, and finally, $\mathbf{l}$ is a constant vector of modulus l directed form point 1 two 2. From familiar rules of triple vectorial product, and after dividing and multiplying with dt we can transform Equation 8.3 to,

$$\mathcal{E}_i = \frac{\mathbf{B} \cdot (\mathbf{l} \times \mathbf{v}\, dt)}{dt} \qquad 8.4$$

From the geometry of the problem, we can rewrite Equation 8.4 as,

$$\mathcal{E}_i = -\frac{\mathbf{B} \cdot \mathbf{n}\, dS}{dt} \qquad 8.5$$

where $\mathbf{n}$ is the unit vector directed out of drawing and dS is the infinitesimal area element swept by the rod during the time interval dt As we see, the nominator is the magnetic flux passing through element dS Finally we arrive to the famous expression for Faraday's law:

$$\mathcal{E}_i = -\frac{d\Phi}{dt} \qquad 8.6$$

As a result, we have derived Faraday's law of induction from more fundamental rules of Physics. Our explanation of the appearance of an induced EMF relates to the case when the magnetic field is constant, while the geometry of the loop changes. The magnetic flux through the loop can also be changed, however, by changing $\mathbf{B}$. In this case, the explanation of the appearance of an EMF will differ in principle. But anyway, the relation between the induced EMF and the changes in the magnetic flux in this case too is described by Eq. (8.6).

Let us calculate the induced voltage on a coil placed inside a solenoid. The magnetic field produced by a sufficiently long cylindrical coil of length L and total number of turns N_1 is

$$B = \frac{\mu_0 N_1 I}{L} \qquad 8.7$$

$\mu_0 = 4\pi \times 10^{-7}\, Vs/Am$ is the magnetic permeability of free space. On the other hand, if an alternating current $I(t) = I_0 \sin(2\pi f t)$ of frequency f is applied to the solenoid, then the resulting magnetic field would also change with time.

In this experiment, time-varying magnetic field in Equation 8.5 will pass through a coil with winding N_2 and cross sectional area of A. Total flux through the secondary coil is $\Phi = BAN_2$. Using Equations 8.6 and 8.7, the induced voltage is calculated as,

$$U = -\frac{\mu_0 A N_1 N_2}{L}\frac{dI}{dt} = -\frac{\mu_0 A N_1 N_2}{L} 2\pi f I_0 \cos(2\pi f t) \quad 8.8$$

However, our measuring devices indicate only effective values of alternating quantities. This effective value is defined as square root of square average of the original signal over one period $T = 1/f$ After straightforward calculation, we find the following relation between measurable quantities:

$$U_e = \frac{2\pi\mu_0 N_1}{L} A N_2 f I_e \quad 8.9$$

In this experiment, we will investigate the effective current strength (I_e), frequency (f), number of winding (N_2) and cross sectional area (A) dependence of induced voltage.