Experiment No **:** EM7

Experiment Name: Magnetic moment of a conducting loop in the magnetic field

Objective: Experimentally observation that the magnetic field may have a torque and thus a magnetic moment.

Keywords: Magnetic force, torque, magnetic dipole moment, Helmholtz coils, magnetic field.

Theoretical Information:

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field directed parallel to the plane of the loop, as shown in Figure 7.1.

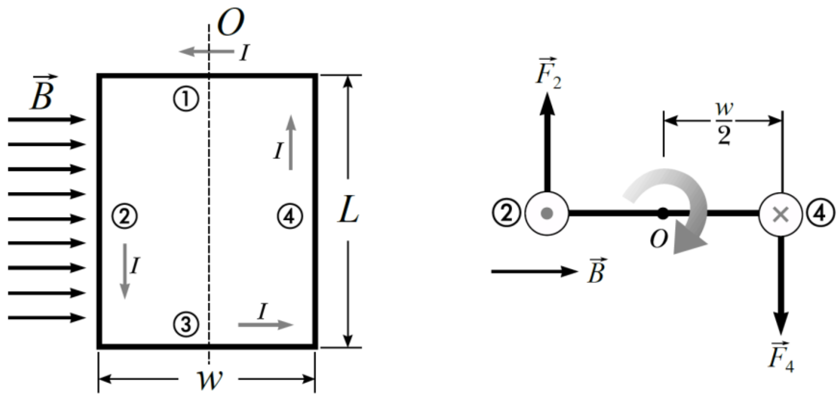

Figure 7.1: Rectangular current loop in a uniform magnetic field.

No magnetic forces act on sides 1 and 3 because these wires are parallel to the field. However, magnetic forces do act on sides 2 and 4 because these sides are oriented perpendicular to the field. The magnitude of these forces is $F_2 = F_4 = ILB$. Considering the right-hand rule, direction of F_2 is out of the page and direction of F_4 is into the page. If the loop is pivoted so that it can rotate about O axis, these two forces produce a torque about O. The magnitude of torque, τ, is

$$\tau = F_2 \frac{w}{2} + F_4 \frac{w}{2} = (ILB)\frac{w}{2} + (ILB)\frac{w}{2} = ILwB \qquad 7.1$$

where the moment arm about O is $w/2$ for each force. Because of the area of the loop is $A = wL$, we can express the torque as

$$\tau = IAB \qquad 7.2$$

Remember that this result is valid only when the magnetic field is parallel to the loop plane. Let's assume that the loop makes a small deviation with the θ angle concerning the O axis and look horizontally from edge 3 (Figure 7.2).

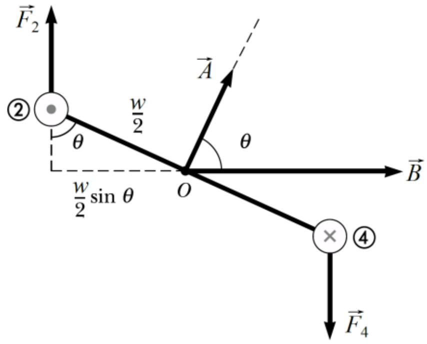

Figure 7.2: View of the loop with a small deviation from the initial position.

In this case, magnitude of the torque is

$$\tau = F_2\frac{w}{2}sin\theta + F_4\frac{w}{2}sin\theta = (ILB)\frac{w}{2}sin\theta + (ILB)\frac{w}{2}sin\theta = IwLBsin\theta \qquad 7.3$$

$$\tau = IABsin\theta \qquad 7.4$$

This is the most general formula that gives the magnitude of the torque for the system under consideration. Transforming above expression to vector quantities, the torque acting on a current loop in a uniform magnetic field can be written as

$$\vec{\tau} = I\vec{A}\times\vec{B} \qquad 7.5$$

where $\vec{A}$, the vector shown in Figure 7.3, pointing out of plane of the circuit and having a magnitude equal to the area of the same. We determine the direction of $\vec{A}$ using the right-hand rule described in Figure 7.3. When you curl the fingers of your right hand in the direction of the current in the loop, your thumb points in the direction of $\vec{A}$.

Figure 7.3: Direction of the magnetic dipole moment of the current loop.

The product $I\vec{A}$ is defined as the magnetic dipole moment, $\vec{\mu}$, of the loop:

$$\vec{\mu} = I\vec{A} \qquad 7.6$$

The SI unit of magnetic dipole moment is amper-meter^2 (A·m^2). Using this definition, we can express the torque as follow,

$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad 7.7$$

Note that this result is analogous to $\vec{\tau} = \vec{p} \times \vec{E}$, for the torque exerted on an electric dipole in the presence of an electric field $\vec{E}$, where $\vec{p}$ is the electric dipole moment. If a coil of wire contains N loops of the same area, the magnetic moment $\vec{\mu}$ of the loop reads

$$\vec{\mu} = NI\vec{A} \qquad 7.8$$

In the experiment, the magnetic torque will be measured depending on the current. The current passing through the Helmholtz coils will cause the magnetic field, which induces a torque on the current loop, that causes it to rotate. In order to visualize this rotation, a mirror will be attached to the current loop which reflects the light incident from a steady source placed across the coil and above the ruler screen. From the geometry of the setup, angular deflection (α) and position of the light spot on the ruler (x) can be derived as

$$\alpha = \frac{x}{L} \qquad 7.9$$

where L is the mirror-ruler distance. If we want to find an explicit theoretical expression (Equation 7.7) for magnetic torque specific to our setup, one would easily find,

$$\tau_{mag} = INA \cdot \kappa I_H \cdot \sin\alpha \qquad 7.10$$

Here I_H is current passing through Helmholtz coils and κ is proportionality coefficient between I_H and magnetic field generated by coils.

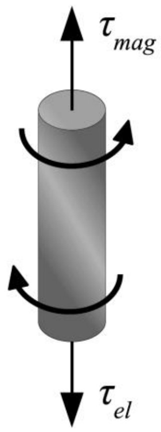

Figure 7.4: Effect of τ_{mag} and τ_{el} on the wire is shown schematically.

As the loop is attached to a vertically pinched wire of finite torsional stiffness, magnetic torque will be balanced by an elastic torque of

$$\tau_{el} = D\alpha \qquad 7.11$$

where $D \approx 3.09 \times 10^{-4}\ N{\cdot}m/rad$ is a constant specific to the experimental setup. In the state of balance, two torque expressions become equal: $\tau_{mag} = \tau_{el}$. Finally, if we substitute $\alpha = x/L$ in τ_{el} we would find the torque formula which you are supposed to use in experimental part.