Experiment No : EM5

Experiment Name: Magnetic Fields of Solenoids / Biot-Savart Law

Objective: Establish a relationship for how the magnetic field of a solenoid varies with current, distance and number of turns per unit length by using the Biot-Savart law.

1. The dependence of the magnetic field on the current passing through the solenoids:
2. The dependence of the magnetic field on the number of turns per unit length of solenoid:
3. The dependence of the magnetic field on the distance from center of the solenoid:

Keywords: Biot-Savart law, Ampere law, magnetic field, current, solenoid.

Theoretical Information:

A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire–which we shall call the interior of the solenoid–when the solenoid carries a steady current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

Figure 5.1 (l*eft*) shows the magnetic field lines surrounding a loosely wound solenoid. Note that the field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is strong and almost uniform.

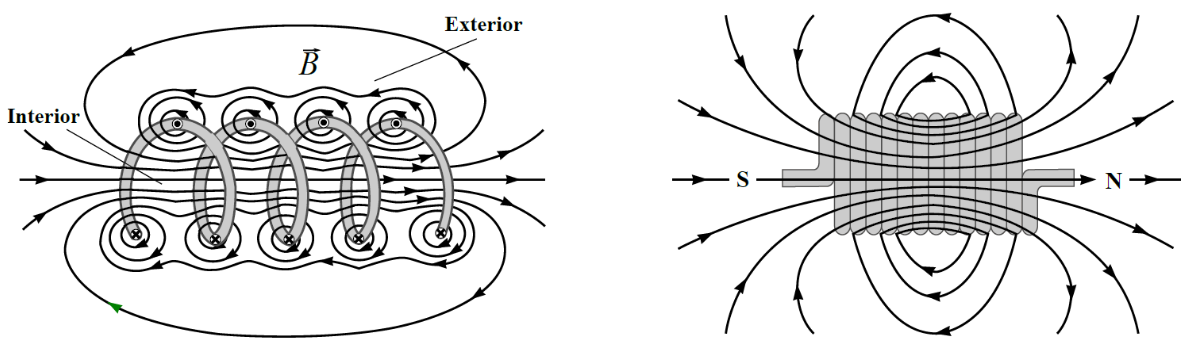

Figure 5.1: (*Left*) The magnetic field lines for a loosely wound solenoid. (*Right*) Magnetic field lines for a tightly wound solenoid of finite length.

If the turns are closely spaced and the solenoid is of *finite* length, the magnetic field lines are as shown in Figure 5.1 (*right*). This field line distribution is similar to that surrounding a bar magnet. Hence, one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the south pole.

As the length of the solenoid increases, the interior field becomes more *uniform* and the exterior field becomes weaker. An *ideal* solenoid is approached when the turns are closely spaced and the length is much greater than the radius of the turns ($L \gg R$). Figure 5.2 shows a longitudinal cross section of part of such a solenoid carrying a current I. In this case, the external field is close to zero, and the interior field is uniform over a great volume.

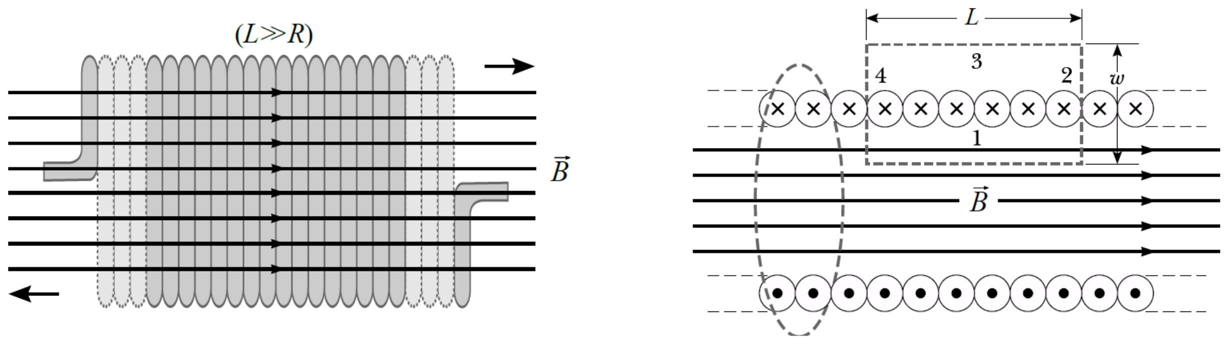

Figure 5.2: (*Left*) Magnetic field lines for a tightly wound ideal solenoid ($L \gg R$) of infinite length, carrying a steady current. (*Right*) Cross-sectional view of an ideal solenoid.

We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in an *ideal solenoid*. Because the solenoid is ideal, $\vec{B}$ in the interior space is uniform and parallel to the axis, and the magnetic field lines in the exterior space form circles around the solenoid. The planes of these circles are perpendicular to the page.

The general case, known as Ampère's law, can be stated as follows: The line integral of $\vec{B}\cdot d\vec{s}$ around any closed path equals $\mu_0 I_{encl}$, where I is the total *enclosed* steady current passing through any surface bounded by the closed path.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl} \tag{5.1}$$

Consider the rectangular (*an amperian loop*) path of length L and width w shown in Figure 5.2 (*right*). We can apply Ampère's law to this path by evaluating the integral of $\vec{B}\cdot d\vec{s}$ over each side of the rectangle. The contribution along side 3 is zero because the magnetic field is zero outside the solenoid. The contributions from sides 2 and 4 are both zero, because $\vec{B}$ is perpendicular to $d\vec{s}$ along these paths, both inside and outside the solenoid. Side 1 gives a contribution to the integral because along this path $\vec{B}$ is uniform and parallel to $d\vec{s}$, that is, the only contribution comes from path 1.

$$\oint \vec{B}\cdot d\vec{s} = \int_{path\ 1}^{B\parallel ds} \vec{B}\cdot d\vec{s} + \int_{path\ 2}^{B\perp ds} \vec{B}\cdot d\vec{s} + \int_{path\ 3}^{B=0} \vec{B}\cdot d\vec{s} + \int_{path\ 4}^{B\perp ds} \vec{B}\cdot d\vec{s} = BL = \mu_0 I_{encl}$$

If N is the number of turns in the length L, the total current through the rectangle is NI. Therefore, Ampère's law applied to this path gives

$$\oint \vec{B}\cdot d\vec{s} = BL = \mu_0 NI \Longrightarrow B = \mu_0 \frac{N}{L} I = \mu_0 n I \tag{5.2}$$

where $n = N/L$ is the number of turns per unit length.

For infinitely long solenoids ($L \gg R$), the magnetic field inside the solenoid is given by Equation 5.2, and is constant along the axis of the solenoid. However, the magnetic field depends on the position of the point on axis of the finite size solenoid ($L \sim R$). When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

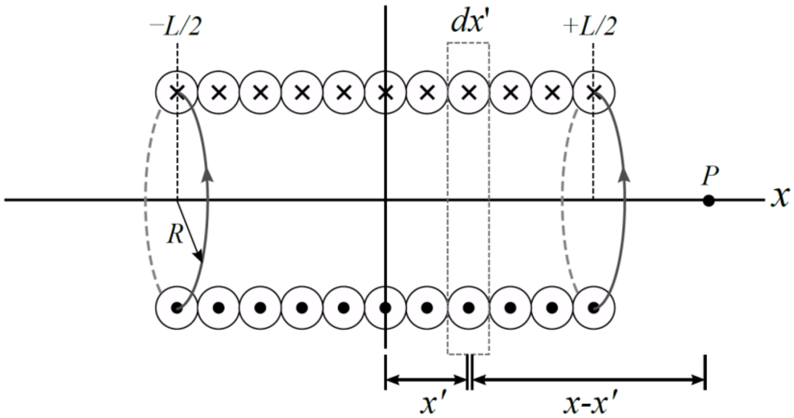

Figure 5.3: Finite size solenoid.

Consider a section of the solenoid of length dx'. The total current winding around the solenoid in that section is $dI = nIdx'$. This section is located at a distance $x - x'$ away from the point P. The contribution to the magnetic field at P due to this subset of loops is

$$dB_x = \frac{\mu_0 R^2}{2[(x-x')^2 + R^2]^{3/2}} dI = \frac{\mu_0 R^2}{2[(x-x')^2 + R^2]^{3/2}} nIdx' \qquad 5.3$$

Integrating over the entire length of the solenoid, we obtain

$$B_x = \frac{\mu_0 n I R^2}{2} \int_{-L/2}^{+L/2} \frac{dx'}{[(x-x')^2 + R^2]^{3/2}} = \frac{\mu_0 n I R^2}{2} \frac{(x-x')}{R^2[(x-x')^2 + R^2]}\Bigg|_{-L/2}^{+L/2}$$

$$= \frac{\mu_0 n I}{2}\left[\frac{x + L/2}{\sqrt{(x+L/2)^2 + R^2}} - \frac{x - L/2}{\sqrt{(x-L/2)^2 + R^2}}\right] \qquad 5.4$$

Equation 5.4 expresses the change of magnetic field depending on the distance from the center.

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[1] Raymond A. Serway, John W. Jewett, Physics for Scientists and Engineers, 6th ed., Thomson Brooks/Cole (2004).