**Experiment No : ** EM4

Experiment Name: Magnetic Fields of Single Coils / Biot-Savart's Law

Objective: Establish a relationship for how the magnetic field of a circular conducting loops varies with current, radius and distance from the axis by using the Biot-Savart law.

1. The dependence of the magnetic field on the current passing through the conducting ring,
2. The dependence of the magnetic field on the radius of the conducting rings,
3. The dependence of the magnetic field on the distance from center of the conducting ring.

Keywords: Biot-Savart law, magnetic field, current, conducting circular loop.

Theoretical Information:

We have seen that mass produces a gravitational field and also interacts with that field. Charge produces an electric field and also interacts with that field. Since moving charge (that is, current) interacts with a magnetic field, we might expect that it also creates that field─and it does.

The equation used to calculate the magnetic field produced by a current is known as the Biot-Savart law. It is an empirical law named in honor of two scientists Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) who investigated the interaction between a straight, current-carrying wire and a permanent magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field.

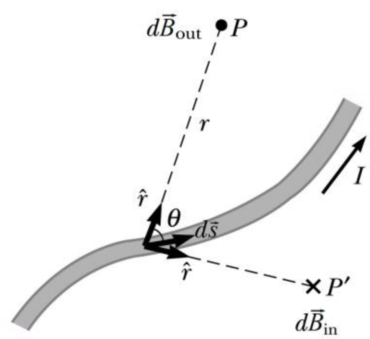

Figure 4.1: The magnetic field $d\vec{B}$ at a point due to the current I through a length element $d\vec{s}$.

This law enables us to calculate the magnitude and direction of the magnetic field produced by a steady current I in a wire. The Biot-Savart law states that at any point P, the magnetic field $d\vec{B}$ due to an element $d\vec{s}$ of a current-carrying wire is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{s} \times \hat{r}}{r^2} \qquad 4.1$$

where, $\hat{r}$ is the unit vector and $d\vec{s}$ is a vector with length ds, in the same direction as the current in the conductor. The constant $\mu_0$is known as the permeability of free space and is exactly

$$\mu_0 = 4\pi \times 10^{-7} = 1.26 \times 10^{-6}\ \text{T·m/A}$$

in the SI system.

Equation 4.1 is called the Biot-Savart law. Note that the field $d\vec{B}$ in Equation 4.1 is the field created by the current in only a small length element $d\mathbf{s}$ of the conductor. To find the total magnetic field $\vec{B}$ created at some point by a current of finite size, we must sum up contributions from all current elements $I\, d\mathbf{s}$ that make up the current. That is, we must evaluate $\vec{B}$ by integrating Equation 4.1:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{s} \times \hat{r}}{r^2} \qquad 4.2$$

where the integral is taken over the entire current distribution. This expression must be handled with special care because the integrand is a cross product and therefore a vector quantity. We shall see one case of such an integration in following *circular wire loop*.

Consider a circular wire loop of radius R located in the yz plane and carrying a steady current I, as in Figure 4.2. We can use the Biot-Savart law to find the magnetic field due to the current I at an axial point P a distance x from the center of the loop

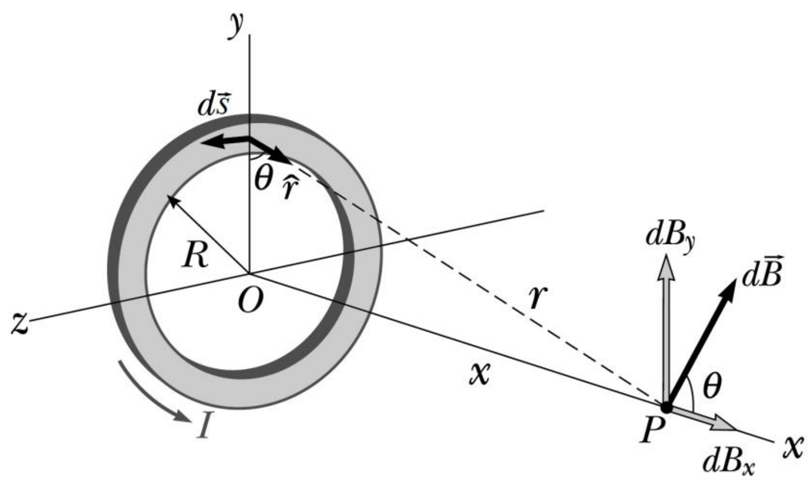

Figure 4.2: Geometry for calculating the magnetic field at a point P lying on the axis of a current loop.

In this situation, every length element $d\mathbf{s}$ is perpendicular to the vector $\hat{r}$ at the location of the element. Thus, for any element, $|d\vec{s} \times \hat{r}| = ds \cdot 1 \cdot \sin\left(\frac{\pi}{2}\right) = ds$. Furthermore, all length elements around the loop are at the same distance r from P, where $r^2 = x^2 + R^2$. Hence, the magnitude of $d\vec{B}$ due to the current in any length element ds is

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(x^2 + R^2)} \qquad 4.3$$

The direction of $d\vec{B}$ is perpendicular to the plane formed by $\hat{r}$ and $d\vec{s}$, as shown in Figure 4.2. We can resolve this vector into a component dB_x along the x axis and a component dB_y perpendicular to the x axis. When the components dB_y are summed over all elements around the loop, the resultant component is zero. That is, by symmetry the current in any element on one side of the loop sets up a perpendicular component of $d\vec{B}$ that cancels the perpendicular component set up by the current through the element diametrically opposite it. Therefore, the resultant field at P must be along the x axis and we can find it by integrating the components $dB_x = dBcos\theta$. That is, $\vec{B} = B_x \hat{\imath}$ where

$$dB_x = \oint dBcos\theta = \frac{\mu_0 I}{4\pi} \oint \frac{ds\ cos\theta}{x^2 + R^2} \quad 4.4$$

and we must take the integral over the entire loop. Because θ, x, and R are constants for all elements of the loop and because $cos\theta = R/(x^2 + R^2)^{1/2}$, we obtain

$$B_x = \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{3/2}} \oint ds = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}} \quad 4.5$$

where we have used the fact that $\oint ds = 2\pi R$ (the circumference of the loop).

To find the magnetic field at the center of the loop, we set $x = 0$ in Equation 4.5. At this special point, therefore,

$$B = \frac{\mu_0 I}{2R} \quad 4.6$$

The pattern of magnetic field lines for a circular current loop is shown in Figure 4.3. For clarity, the lines are drawn for only one plane−one that contains the axis of the loop.

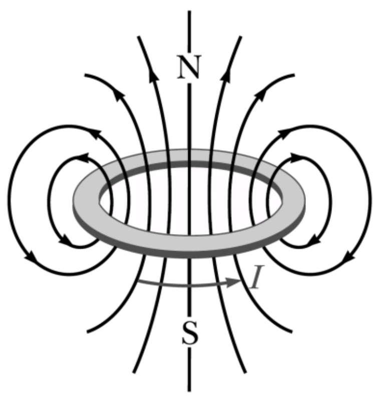

Figure 4.3: Magnetic field lines surrounding a current loop.

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[1] Raymond A. Serway, John W. Jewett, Physics for Scientists and Engineers, 6th ed., Thomson Brooks/Cole (2004).