Experiment No : EM2

Experiment Name: Dielectric constant of different materials

Objective:

1. The relation between charge Q and voltage U is to be observed using a plate capacitor. The electric constant ε_0 is to be determined from the relation between charge Q and voltage U.
2. Determination of dielectric constant of the dielectric (plastic) media is introduced between the plates of a capacitor.
3. Observation of the variation in the amount of charge deposited on the capacitor plates with a dielectric material.

Keywords: Electric constant, capacitance of a plate capacitor, dielectric constant

Theoretical Information:

Electrostatic processes in vacuum (and with a good degree of approximation in air) are described by the following integral form of Maxwell's equations:

$$\phi_E = \oiint_A \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \qquad 2.1$$

$$\oint \overrightarrow{E} \cdot d\vec{S} = 0 \qquad 2.2$$

where $\vec{E}$ is the electric field intensity, Q the charge enclosed by the closed surface A, ε_0 the electric constant, and S a closed path.

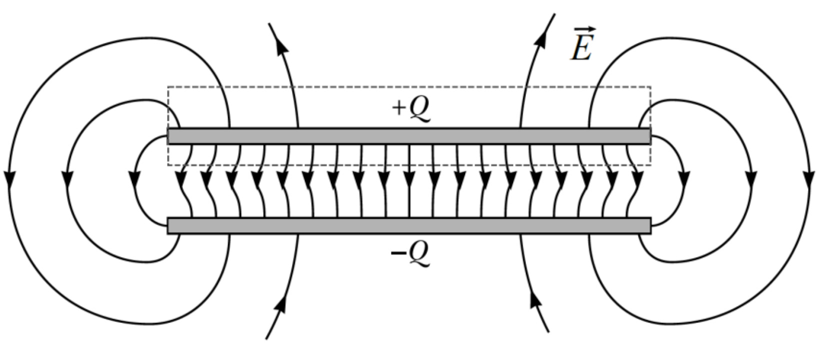

Figure 2.1: Electric field of a plate capacitor with small distance between the plates, as compared to the diameter of the plates. The dotted lines indicate the volume of integration.

If a voltage U is applied between two capacitor plates, an electric field will $\vec{E}$ prevail between the plates, which is given by:

$$U = -\int_1^2 \vec{E} \cdot d\vec{r} \qquad 2.3$$

Due to the electric field, electrostatic charges of the opposite sign are drawn towards the surfaces of the capacitor. Voltage source does not generate any charge. The charges are separated only. Therefore, the absolute values of the opposite electrostatic induction charges must be equal.

Assuming the electric field lines will, always, be perpendicular to the capacitor surfaces of area *A*, we can obtain the following equation from Equation 2.1

$$\frac{Q}{\varepsilon_0} = \vec{E} \cdot \vec{A} = UA\frac{1}{d} \qquad 2.4$$

which can be experimentally verified for small distances *d* between the capacitor plates. The ratio of the charge on a capacitor and the potential difference between its plates is constant. This constant value is called as *capacitance* of a capacitor. The quantity of charge Q on a capacitor is linearly proportional to the potential difference between the conductors; that is, $Q \propto U.$ The proportionality constant depends on the shape and separation of the conductors. We can write this relationship as $Q = CU$ if we define capacitance as follows: The capacitance *C* of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors: $C = Q/U$.

$$Q = CU = \varepsilon_0\frac{A}{d}U \qquad 2.5$$

The linear relationship between charge *Q* and voltage *U* is given by Equation 2.5. For constant voltage, the inverse distance between the plates, and thus the capacitance, is a measure for the amount of charge a capacitor can take. Equation 2.6, also, shows that the capacitance *C* of a parallel-plate capacitor is proportional to the area *A* of its plates and inversely proportional to the plate separation *d*

$$C = \varepsilon_0 A\frac{1}{d} \qquad 2.6$$

Inversely, if *U*, *Q*, *d*, and *A* are measured, the electric constant ε_0 can be calculated:

$$\varepsilon_0 = \frac{d}{A}\frac{Q}{U} \qquad 2.7$$

Equations 2.5, 2.6, and 2.7 are valid only approximately, due to the assumption that field lines are parallel. With increasing distances between the capacitor plates, electric field becomes non-uniform and causes a large electric constant according to Equation 2.7. This is why the value of the electric constant ε_0 should be determined for a small and constant distance between the plates.

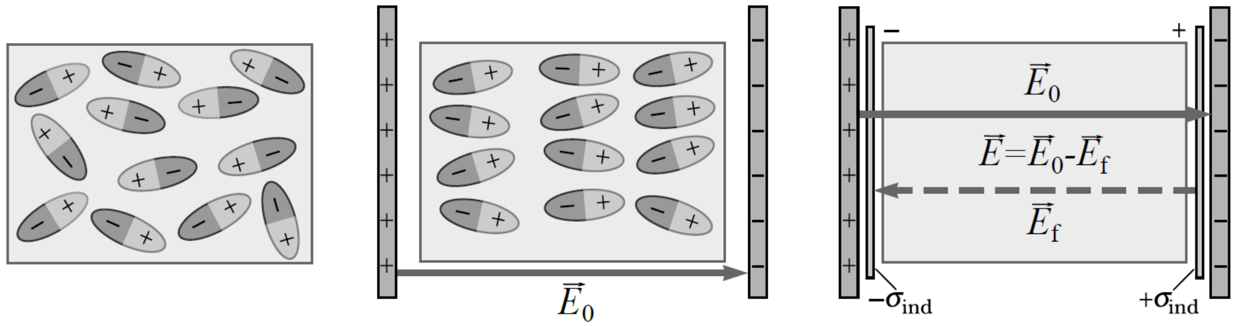

Figure 2.2: Generation of free charges in a dielectric through polarization of the molecules in the electric field of a plate capacitor.

These changes when insulating materials (*dielectrics*) are inserted between the plates. Dielectrics have no free moving charge carriers, as metals have, but they have positive nuclei and negative electrons. These may be arranged along the lines of an electric field. Molecules that were previously nonpolar behave as locally stationary dipoles. As shown in Figure 2.2, the effects of the single dipoles cancel each other macroscopically inside the dielectric. However, no partners with opposite charges are present on the surfaces; they have stationary charges, called free charge (Q_f). The free charges, in turn, weaken the electric field $\overrightarrow{E_0}$ of the real charges Q, which are on the capacitor plates, within the dielectric. The weakening of the electric field $\overrightarrow{E_0}$ within the dielectric is expressed by the dimensionless, material specific dielectric constant ε ($\varepsilon = 1$ in vacuum):

$$\vec{E} = \frac{\vec{E}_0}{\varepsilon} \qquad 2.8$$

Where $\overrightarrow{E_0}$ is the electric field generated only by the real charges Q. Thus, the opposite field generated by the free charges must be:

$$\vec{E}_f = \overrightarrow{E_0} - \vec{E} = \frac{\varepsilon - 1}{\varepsilon}\vec{E}_0 \qquad 2.9$$

Neglecting the charges within the volume of the dielectric macroscopically, only the free surface charges ($\pm Q_f$) generate the opposite field effectively:

$$\overrightarrow{E_f} = \frac{Q_f}{A\varepsilon_0} = \frac{Q_f d}{V\varepsilon_0} = \frac{p}{V\varepsilon_0} \qquad 2.10$$

where p is the total dipole moment of the surface charges. In the general case of an inhomogeneous dielectric, Equation 2.10 becomes:

$$\vec{E}_f = \frac{1}{\varepsilon_0}\int \frac{d\vec{p}}{dV} = \frac{1}{\varepsilon_0}\vec{P} \qquad 2.11$$

where $\vec{P}$, – total dipole moment per unit volume – is called dielectric polarization. If additionally a $\vec{D}$ -field (dielectric displacement) is defined:

$$\vec{D} = \varepsilon\varepsilon_0\vec{E} \qquad 2.12$$

the field lines only begin or end in real (directly measurable) charges, the three electric magnitudes, which are field intensity $\vec{E}$, dielectric displacement $\vec{D}$, and dielectric polarization $\vec{P}$, are related to one another through the following equation:

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon\varepsilon_0\vec{E} \qquad 2.\ 13$$

If the real charge Q remains on the capacitor, whilst a dielectric is inserted between the plates, voltage $U_{dielectic}$ between the plates is reduced as compared to voltage U_{air} in the air by the dielectric constant:

$$U_{dielectic} = \frac{U_{air}}{\varepsilon} \qquad 2.14$$

Similarly, one obtains from the definition of capacitance from Equation 2.4:

$$C_{dielectric} = \varepsilon C_{air} \qquad 2.15$$

The general form of Equation 2.5 is thus:

$$Q = \varepsilon\varepsilon_0\frac{A}{d}U \qquad 2.16$$

If the charges obtained with and without dielectric, Equations 2.5 and 2.16, are divided by each other, the obtained numerical value is the dielectric constant of the material.

$$\varepsilon_{dielectric} = \frac{Q_{dielectric}}{Q_{air}} \qquad 2.17$$

In order to take into consideration the above described influence of free charges, Maxwell's equation 1 is generally completed by the dielectric constant of the dielectric which fills the corresponding volume:

$$\oint\!\!\oint_A \varepsilon\varepsilon_0\vec{E}\cdot d\vec{A} = \oint\!\!\oint \vec{D}\cdot d\vec{A} = Q \qquad 2.18$$

Thus, Equation 2.16 becomes Equation 2.4.