Experiment No : EM1

Experiment Name: Capacitance of metal spheres and of a spherical capacitor

Objective:

1. Determination of the capacitance of two metal spheres with different diameters.
2. Determination of the capacitance of a spherical capacitor.

Keywords: Potential, electric charge, electric field, capacitance, capacitor

Theoretical Information:

The circuit element for charge storage is called as a *capacitor*. The capacity of a capacitor is called *capacitance*. Capacitance is the ability to store electric charge. Every object that can be charged with electricity has a capacity.

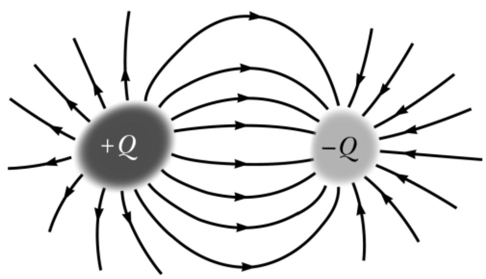

Figure 1.1: A simple capacitor

Consider two conductors carrying charges of equal magnitude and opposite sign, as shown in Figure 1.1. Such a combination of two conductors is called a capacitor. A potential difference ΔV exists between the conductors due to the presence of the charges. What determines how much charge is on the plates of a capacitor for a given voltage? Experiments show that the quantity of charge Q on a capacitor is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$ The proportionality constant depends on the shape and separation of the conductors. The capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference. We can write this relationship as $Q = C\Delta V$ if we define capacitance as follows: The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors: $C = Q/\Delta V$ Therefore, capacitance is a measure of a capacitor's ability to store charge. In other words, the capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

The SI unit of capacitance is the **farad** (F), which was named in honor of Michael Faraday. [1 Farad = Coulomb/Volt] The farad is a very large unit of capacitance. In practice, typical devices have capacitance ranging from (1μF =10^{-6} F), nanofarad (1nF =10^{-9} F) to pikofarad (1pF =10^{-12} F).

In electrical circuits, the term *capacitance* is usually a shorthand for the *mutual capacitance* between two adjacent conductors, such as the two parallel plates of a capacitor, or a spherical capacitor with two concentric spherical shells. However, for an *isolated* conductor there also exists a property called *self-capacitance*, which is the amount of electric charge that must be added to an isolated conductor to raise its electric potential by one unit.

For example, consider an isolated charged spherical conductor. The electric field lines around this conductor are exactly the same as the shell system, which is concentric with the same sphere with an infinite radius and carries an equal-magnitude, but opposite-labeled charge. Thus, we can imagine that this sphere is the second conductor of a two-conductor concealed imaginary shell. The capacitance for this case is calculated as follows

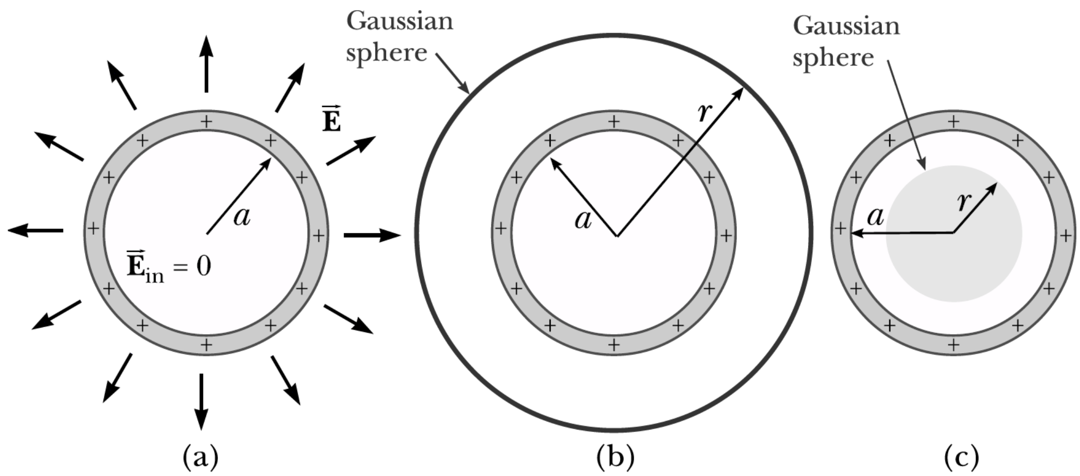

Figure 1.2: Representation of Gaussian spheres drawn for a conductive sphere

The electric field is constant and perpendicular to the surface of the sphere at each point, thus the electric field vector $\vec{E}$ and the area vector $d\vec{A}$ are parallel to each other. By taking the integral over the entire surface, we can write the electric flux as: $\phi_E = \vec{E} \cdot \vec{A} = EA$ on the surface. If we apply the Gauss's law for the spherical Gaussian surface for radius $r<a$, which is concentric with the conductive spherical shell as shown in Fig. 1.2(c), we can see from Equation 1.1 that the electric field is equal to zero for $r<a$, since there is no charge in the conductive sphere.

$$\phi_E = \vec{E} \cdot \vec{A} = EA = \frac{q_{enc}}{\varepsilon_0} = 0 \qquad 1.1$$

On the other hand, because of the spherical symmetry of the charge distribution, the field outside is the same as, due to a point charge Q located at the center of the shell. Therefore, outside of the conductive sphere, r > a, the electric field flux is;

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\, dA = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \qquad 1.2$$

The electric field is constant and perpendicular to the surface of the sphere at each point, thus the electric field vector $\vec{E}$ and the area vector $d\vec{A}$ are parallel to each other. By taking the integral over the entire surface, we can write the electric flux as:

$$\phi_E = \oint \vec{E}\, d \cdot \vec{A} = E \oint dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad 1.3$$

We can calculate the potential difference ΔV from the electric field E obtained from Equation 1.3;

$$\Delta V = -\int_R^\infty \vec{E} \cdot d\vec{r} = -\int_R^\infty \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \cdot d\vec{r} = -\frac{Q}{4\pi\varepsilon_0}\int_R^\infty r^{-2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\infty} - \frac{1}{R}\right] = \frac{Q}{4\pi\varepsilon_0 R} \qquad 1.4$$

If this expression is substituted in the capacitance expression, the capacitance of an isolated charged spherical conductor shell is;

$$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0 R \qquad 1.5$$

The potential difference ΔV of a spherical capacitor can be calculated by changing the boundaries of the integral in Equation 1.4. If the calculation is repeated with appropriate boundary condition, the result is;

$$\Delta V = -\int_{R_2}^{R_2} \vec{E} \cdot d\vec{r} = -\int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \cdot d\vec{r} = -\frac{Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2} r^{-2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_1}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right] \qquad 1.6$$

If the expression received from Equation 1.6 is substituted into $C = \frac{Q}{\Delta V}$, for a spherical capacitor, the capacitance is;

$$C = 4\pi\varepsilon_0\left[\frac{R_1 R_2}{R_2 - R_1}\right] \qquad 1.7$$

The results show that the capacitance of an isolated charged sphere and a spherical capacitor are proportional to its radius and the capacitance is independent of both charge Q on the sphere and the potential difference ΔV.