Experiment No : M4

Experiment Name: Hooke's Law

Objective:

1. Determining the change of length ΔL of two helical springs with different turn diameters as a function of the gravitational force F exerted by the suspended weights.
2. Confirming Hooke's law and determining the spring constants k of the two helical springs.

Keywords: Hooke's law, spring constant, oscillation, period.

Theoretical Information:

Holding a spring in either its compressed or stretched position requires that someone or something exerts a force on the spring. This force is directly proportional to the displacement, Δx, of the spring. In turn, the spring will exert an equal and opposite force

$$F = -k\Delta x \qquad 10.1$$

where k is called the "spring constant." This is often referred to as a "restoring force" because the spring exerts a force in the direction opposite to the displacement, indicated by the negative sign. The Eq. 10.1 is known as Hooke's law.

Simple harmonic motion will occur whenever there is a restoring force that is proportional to the displacement from equilibrium, as is in Hooke's law. From Newton's second law, $F = ma$, and recognizing that the acceleration a is the second derivative of displacement with respect to time. The classical equation of motion for a one-dimensional simple harmonic oscillator with a particle of mass m attached to a spring having spring constant k is the Eq. 10.1 can be rewritten as;

$$F = -kx \Rightarrow m\frac{d^2x}{d^2t} = -kx \qquad 10.2$$

which can be written in the standard wave equation form;

$$m\frac{d^2x}{d^2t} + kx = 0 \qquad 10.3$$

The Eq. 10.3 is a linear second-order differential equation that can be solved by the standard method of factoring and integrating. The resulting solution to Eq. 10.3 is

$$x(t) = x_0\sin(\omega t + \phi) \qquad 10.4$$

where x_0 is the amplitude of oscillation and

$$\omega = \sqrt{\frac{k}{m}} \qquad 10.5$$

The equations in the form of Eq. 10.4 describe what is called *simple harmonic motion*. The period T, the frequency f, and the constant ω are related by:

$$\omega = 2\pi f = 2\pi/T \qquad 10.6$$

Thus, the period of oscillation T is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad 10.7$$

Note that T does not depend upon the amplitude x_0 of oscillation. Therefore, if a mass is hung from a spring suspended from the vertical, the resulting period of oscillation T would be proportional to the spring constant k and square root of the attached mass m.