

Existence of bell-shaped traveling wave solutions for certain nonlocal wave equations

In this talk I will present some results on the existence of traveling wave solutions $u(x, t) = \varphi(x - ct)$ of a general class of nonlocal wave equations of the form

$$u_{tt} = (\beta * |u|^{p-1} u)_{xx},$$

where $c \in \mathbb{R}$ is the wave velocity, $p > 1$, and $\beta(x)$ is a given bell-shaped (i.e. even, positive and decreasing for $x > 0$) kernel function. These results were obtained in Abba Ibrahim Ramadan's master's thesis (Sabanci University 2015).

Traveling wave solutions is important both in terms of applications, but also theoretically as they are in some sense "critical" points of the solution space. Replacing the form $u = \varphi(x - ct)$, one obtains a time independent equation which in some cases (as the Kortevog De Vries equation) can be solved explicitly. In the general case one has to employ variational methods where the traveling wave appears as a critical point of a certain energy functional. Existence of a critical point usually requires some intricate and technical compactness argument. If possible, restricting to bell-shaped functions simplifies the problem considerably.